Name:________________________________

Recitation Time:________________________

Math 252 **Integral Applications to Physics and Engineering** **Activity** 6

This activity is worth 10 points of course credit. See tentative calendar for due dates. Late activities are accepted at the discretion of your recitation instructor and a penalty may be imposed.

(1) If $F(x)$ is the force in the direction of motion on an object when at a position x (along a line) then the work done by the force on the object in moving the object from $x = a$ to $x = b$ is $W = \int_a^b F(x)\,dx$. Use this to find the work done in each situation below.

(a) A chain that weighs 1 lb per linear foot is connected to a 2 lb pump in a well at a depth of 100 ft (assume the pump is just above the water level in the well). Determine the work (in units of ft-lbs) required to lift the chain and pump out of the well.

Hint: Let x be the distance the pump is above its initial position as it's lifted. So $x = 0$ is the pump at the bottom and $x = 100$ is the pump at the top. Determine the weight force $F(x)$ and integrate!

$W =$

(b) Consider Hooke's Law: A spring exerts a force that is proportional to the amount the spring is compressed or stretched from it's equilibrium position ($F = kx$).

Suppose it takes 10 Newtons of force to compress a spring by 0.5 cm from it's equilibrium position. Determine the work (in Joules (J), 1 J=1 N·m=100 N·cm) required to compress the spring an additional 0.5 cm.

$W =$

(2) The work required to pump the fluid from a tank (between a units and b units above the bottom of a tank) of constant mass-density ρ out to a height h above the bottom of the tank is given by

$$W = \int_a^b \rho g (\text{cross-sectional area at } y)(\text{distance fluid at } y \text{ needs to be lifted})\, dy$$

where g is the acceleration due to gravity and y is the distance from bottom of the tank.

Note: Water has a mass-density of $\rho = 10^3$ kg/m^3 (SI units) and a force-density of $\rho g = 62.4$ lb/ft^3 (in English units). Beware of English units.

(a) A water tank is in the shape of a right-circular cylinder of radius 1 m and height 3 m laying on its side. Assuming the tank is half-full, how much work (in J) is required to pump all the water out to the top of the tank (2 meters above the bottom of the tank).

Hint: Setup a vertical axis so that $y = 0$ is the center of the circular (vertical) cross-section and the positive direction goes down.

$W =$

(b) An inverted right-circular conical gasoline tank of radius 2 ft and height 8 ft is buried in the ground so that the circular top is 1 ft below the ground (parallel to the ground). How much work (in ft-lbs) is required to pump the gasoline occupying the top foot of the tank to a height 2 ft above the ground if the tank is full? (Ignore the water the ends up in the hose from the pumping process after top foot is done being pumped out – assume it's negligible.)

Hint: The force-density of gasoline is $\rho g = 45$ lb/ft^3.

$W =$

(c) An empty 2-meter diameter spherical tank rests on a stand so that the bottom of the tank is 1 m above the ground. How much work is required by a pump in order to fill the tank from the bottom with water (with a hose from ground level)? (Ignore the water the ends up in the hose from the pumping process when the sphere is done being filled – assume it's negligible.)

Hint: The only difference in setting up the integral (when pumping-in instead of pumping-out) comes from determining the distance the fluid is to be pumped.

$W =$

(3) The hydrostatic force on a vertical surface submerged (between a units and b units along some vertical axis – often setup so $a = 0$) in a liquid of constant mass-density ρ is given by

$$F = \int_a^b \rho g(\text{width at } y)(\text{depth of the fluid at } y)\, dy$$

where g is the acceleration due to gravity and y is the variable of the vertical axis.

Note: Water has a mass-density of $\rho = 10^3$ kg/m^3 (SI units) and a force-density of $\rho g = 62.4$ lb/ft^3 (in English units). Beware of English units.

(a) The Dam-It Toy Company builds toy dams. The largest model they have has a lower edge that is in the shape of the curve $y = |\sin(\pi x)|$ from $x = -0.5$ to $x = 0.5$ and an upper edge that is the horizontal line segment joining $(-0.5, 1)$ and $(0.5, 1)$. Here, both x and y are in meters. Setup an integral (BUT DO NOT EVALUATE) that would determine the force (in SI units) dam must withstand due to water behind it from the bottom to within 10 centimeters of the top. You may leave ρ and g symbolic.

$$F = \int$$

(b) A glass viewing window in an above-ground concrete pool at a marine amusement park is in the shape of a circle of radius 2 ft set so that the bottom of the window is 4 ft from the bottom of the pool. Calculate the force (in lbs) the glass must withstand if the concrete pool is filled to a depth of 12 ft.

Hint: Let $y = 0$ be the center of the window (not the bottom of the pool) and integrate from $y = -2$ to $y = 2$.

$F =$