1) Given the matrix:

$$A = \begin{pmatrix} 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ -1 & -1 & -2 & -1 & -2 \\ 1 & 1 & 2 & 1 & 2 \end{pmatrix}.$$

a) Determine all eigenvalues and eigenspaces of the endomorphism $\Phi : \mathbb{Q}^5 \to \mathbb{Q}^5$, $x \mapsto Ax$.

b) Show that $\ker(\Phi) = \ker(\Phi^2)$.

c) Furthermore determine a basis of an Φ-invariant complement to $\ker(\Phi^2)$.