MATH 106 COMPREHENSIVE COMPETENCY ASSESSMENT   (FINAL EXAM)

This is an open-book assessment. You may refer to LEO classroom materials as you work on the exam, and you may use calculator/graphing/spreadsheet software. **You must complete the exam individually. Neither collaboration nor consultation with others is allowed. Use of instructors' solutions manuals or online problem solving services in NOT allowed.**

Record your answers and work on the separate answer sheet provided.

There are 25 problems.
Problems #1–12 are Multiple Choice.
Problems #13–15 are Short Answer. (Work not required to be shown)
Problems #16–25 are Short Answer with work required to be shown.

MULTIPLE CHOICE

1. – 2. "A Place for Fido" is a business that manufactures and sells custom-built dog houses. A small dog house requires 15 square feet (ft^2) of plywood and 6 ft^2 of insulation. A large dog house requires 60 ft^2 of plywood and 18 ft^2 of insulation. The business currently has 900 ft^2 of plywood and 120 ft^2 of insulation on hand. If a small dog house sells for \$55 and a large dog house sells for \$100, what combination of small dog houses ("x") and large dog houses ("y") should be built to maximize the revenue R while staying within constraints?

    1. Identify the building material constraint for insulation:

                                              1. _____

        A. $15x + 60y \geq 900$ C. $6x + 18y \leq 120$

        B. $15x + 60y \leq 900$ D. $6x + 18y \geq 120$

    2. State the objective function.

                                                2. _____

        A. $R = 15x + 60y$ C. $R = 6x + 18y$

        B. $R = 100x + 55y$ D. $R = 55x + 100y$

3. Which of the following statements is **NOT** true:

                                                3. _____

  A.   If an event cannot possibly occur, then the probability of the event is 0.
  B.   If events E and F are mutually exclusive events, then $P(E \cap F) = \emptyset$.
  C.   If only two outcomes are possible for an experiment, then the sum of the probabilities of the outcomes is equal to 1
  D.   Conditional probability is defined as
$$P(A|B) = \frac{P(A \cap B)}{P(A)}$$

4. The Tylers buy a new family home for $299,000, make a down payment of 20%, and finance the rest with a 30-year fixed mortgage at an annual interest rate of 3.9% compounded monthly. What is the amount of their monthly mortgage payment to amortize the loan?

4. _____

A. $1128.21   C. $1127.58
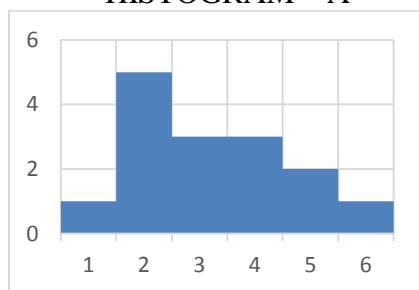
B. $1128.23   D. $1136.46

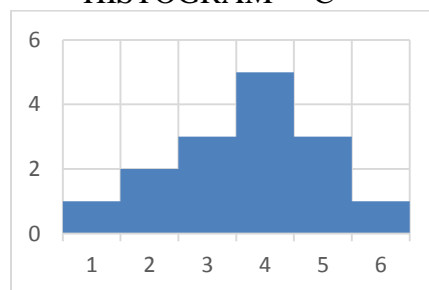5. Which histogram below accurately reflects 15 survey responses presented in the following frequency table?

Value	Frequency
1	X
2	XXXXX
3	XXX
4	XXX
5	XX
6	X

5. _____

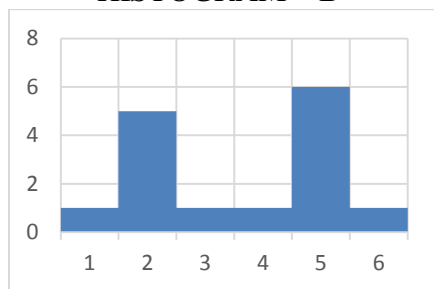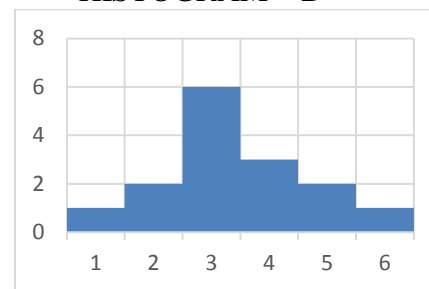HISTOGRAM   A

HISTOGRAM   C
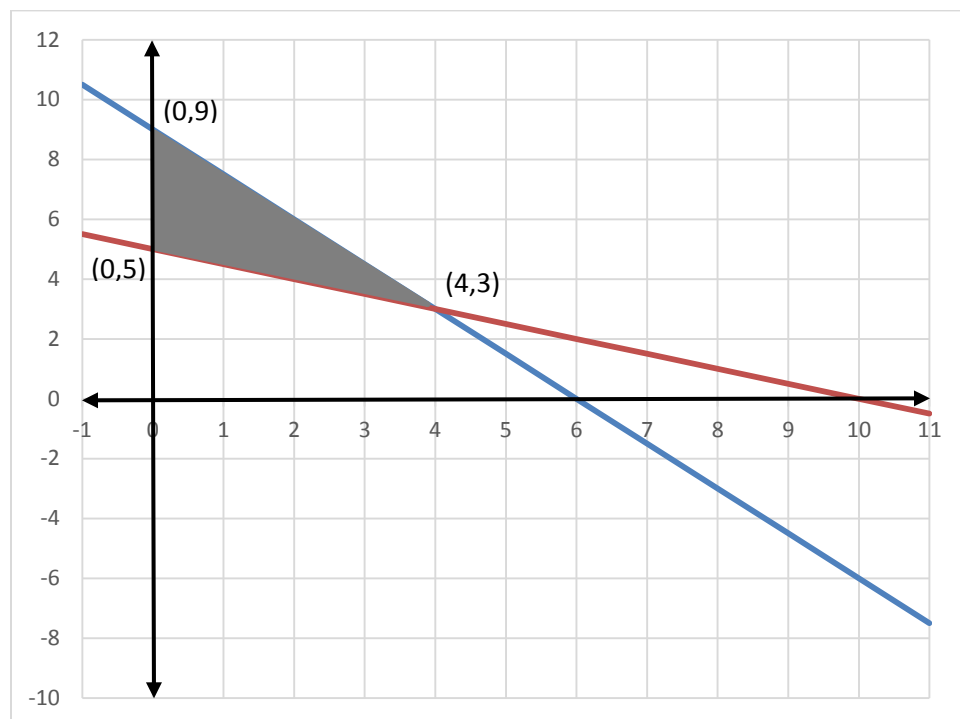
HISTOGRAM   B

HISTOGRAM   D

6. The shaded "feasible region" graphed below satisfies which system of linear inequalities?

7. _____

A.	$3x + 2y \geq 18$	$x + 2y \geq 10$	$x \geq 0$	$y \geq 0$
B.	$3x + 2y \leq 18$	$x + 2y \leq 10$	$x \geq 0$	$y \geq 0$
C.	$3x + 2y \geq 18$	$x + 2y \leq 10$	$x \geq 0$	$y \geq 0$
D.	$3x + 2y \leq 18$	$x + 2y \geq 10$	$x \geq 0$	$y \geq 0$

7. Find the equation of the line passing through $(2, 6)$ and $(-3, -1)$:   7. _____

   A. $x - y = -4$ B. $3x + y = 12$ C. $7x - 5y = -16$ D. $x + y = 8$

8. The amount of money you must deposit into an account paying 8% compounded quarterly in order to be able to make quarterly withdrawals of $1000 for the next 4 years can be determined using formula for:

8. _____

   A. Single-payment, simple interest
   B. Single-payment, compound interest
   C. Sequence of payments: present value of an ordinary annuity
   D. Sequence of payments: future value of an ordinary annuity

9. Which of the corner points for the system of linear inequalities graphed below minimizes the objective function $P = 3x + 4y$?

9. _____
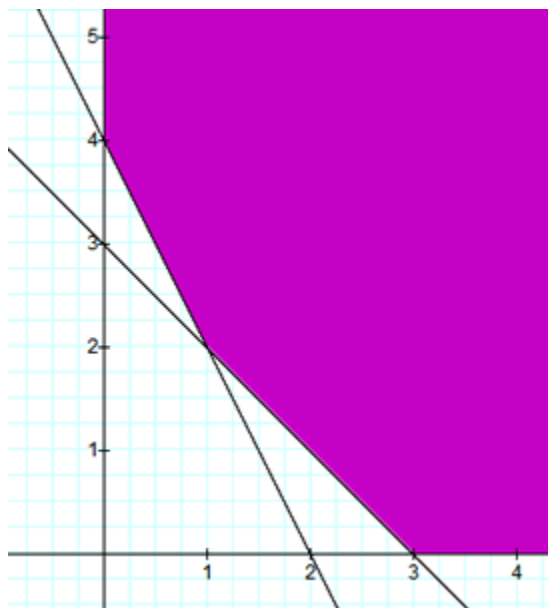
A. (2, 0)   C. (1, 2)

B. (3, 0)   D. (0, 4)

10. The mean time from check-in to completion for a customer served at the Townsburg branch of the Department of Motor Vehicles is 83 minutes, with a standard deviation of 14 minutes. Assuming a normal distribution, what is the probability that a randomly chosen customer experiences service done between 83 and 97 minutes?

10. _____

A. 0.3413   C. 0.5000

B. 0.4772   D. 0.6826

11. Two balls are drawn in succession, without replacement, out of a box containing 5 red and 2 white balls. What is the probability that the first ball drawn is red and the second ball drawn is red?

11. _____

A. $\dfrac{5}{21}$ B. $\dfrac{10}{21}$ C. $\dfrac{2}{3}$ D. $\dfrac{1}{21}$

12. A local bakery is contracted to bake cakes for local grocery stores to sell. The bakery's daily production costs are $1.50 per cake baked plus a fixed cost of $676.   The cakes are sold by the grocery stores for $8.00 each. How many total cakes must be sold daily for the local bakery to break even? *Round answer to the nearest whole cake.*

12. _____

    A. 104   C. 85

    B. 832   D. 72

*   *   *   *   *   *   *   *   *   *   *   *   *

SHORT ANSWER   (work NOT required to be shown)

13. For the linear equation $-7x + 3y = 63$:

    a. Determine the slope: _____

    b. Determine y – intercept if it exists: _____

    c. Express equation in *slope-intercept* form: _____

14. Let $U = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $A = \{1,3,7,11\}$, $B = \{7,13,15,17\}$, and $C = \{1,9,11\}$. List the members of the following sets:

    a. $(B \cap C)$ Answer: _____

    b. $(A \cup C)$ Answer: _____

    c. $(A' \cap B' \cap C')$ Answer: _____

15. "Guilt and focusing on decision making" (Gangemi & Mancini, *Journal of Behavioral Decision Making*, Vol 20, Jan 2007) reported on 171 volunteer students participating in an experiment where each was randomly assigned to one of three groups. One group was made to feel guilty, one group was made to feel angry, and the third group was not influenced. Immediately after reaching these emotional states, the students were asked to decide whether or not to spend lots of money to repair a very old car (not a "historic"/antique). The "stated" option was "spend the money to repair the car". The following *raw data* was recorded:

Emotional State	Choose stated option C	Don't choose stated option C'	Totals
Guilt	45	12	57
Anger	8	50	58
Neutral	7	49	56
Totals	60	111	171

(Report your answers as fractions or as decimal values rounded to the nearest hundredth.)

Find the probability that a randomly-selected student:

   (a) is in the "neutral" emotional state or does NOT choose stated option :   Answer: _____

   (b) does NOT choose stated option, given that student is in the "neutral" state: Answer: _____

   (c) does NOT choose stated option and is in the "neutral" state?   Answer: _____

SHORT ANSWER, with work required to be shown, as indicated.

16. You are a restaurant manager. You must pick 14 entrees for your menu. There are 13 meat offerings and 8 poultry offerings to choose from.

(a) In how many ways can 14 entrees be randomly selected out of the 21 meat and poultry offerings? **Show work.**

(b) In how many ways can 14 entrees be chosen, if 8 must be meat and 6 must be poultry? **Show work.**

(c) If 14 entrees are randomly selected from the 21 offerings, what is the probability that 8 are meat and 6 are poultry? *Round answer to nearest ten-thousandth (4 places after decimal).* **Show work.**

17. Solve the system of equations using substitution, elimination by addition, or augmented matrix methods (your choice). **Show work.**

$$3x - 2y = 7$$
$$2x + 3y = -4$$

18. A freight-hauling firm estimates that it will need a new forklift in six years. The estimated cost of the vehicle is $40,000. The company sets up a sinking fund that pays 8% compounded semiannually, into which it will make semiannual payments to achieve the goal. Calculate the size of the payments. **Show work.**

    A. $2662.09   C. $2107.80

    B. $1023.47   D. $1323.71

19. According to Symantec Corporation, "2016 Internet Security Threat Report" (https://www.symantec.com/security-center/threat-report), "spear-phishing" cyberattacks against American small businesses are steadily increasing. 18% of all "spear-phishing" cyberattacks against American businesses in 2011 targeted "small" businesses (those with less than 250 employees). In 2015, 43% of all "spear-phishing" cyberattacks against American businesses targeted "small" businesses.

(a) Which of the following linear equations could be used to predict annual percentage of all "spear-phishing" cyberattacks against US businesses that target "small" businesses ("y") in a given year "x" since 2011, where $x = 0$ represents the year 2011? **Explain/show work.**

$A.\ \ y = 0.16x + 18$ $\qquad\qquad$ $C.\ \ y = 0.16x + 2011$

$B.\ \ y = 6.25x + 18$ $\qquad\qquad$ $D.\ \ y = 6.25x + 2011$

(b) Use the equation from part (a) to predict the percentage of all "spear-phishing" cyberattacks against US businesses that will target "small" businesses in the year 2019. *Round answer to nearest tenth of a percent.* **Show work.**

(c) Fill in the blanks to interpret the slope of the equation: The rate of change of percent of all "spear-phishing" cyberattacks against US businesses that target "small" businesses with respect to time is _____ per _____. (Include units of measurement.)

20. Veronica's savings account has a balance of $6400. If she makes no deposits or withdrawals for 4 years, and the account accrues interest at 5% compounded quarterly, what will be the total amount of money in her savings account at the end of the 4-year period? *Round answer to the nearest cent*. **Show work**

21. There is a 0.7 probability that MATH 106 students will correctly follow all instructions on the Final Exam. What is the probability that exactly 7 of 10 students in a MATH 106 class section correctly follow all Final Exam instructions? *Round answer to the nearest thousandth (three places after decimal).* **Show work.**

22. The feasible region shown below is bounded by lines $4x - y = 6$, $x + y = 3$, and $y = 0$. Find the coordinates of corner point A. **Show work.**
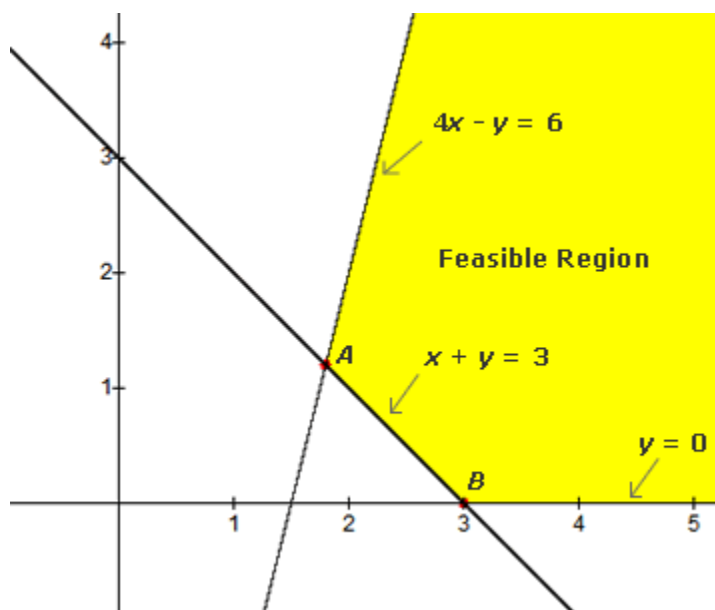
23. A UMUC criminal justice grad works in the Smith County public defenders' office tracking weekly caseload to identify trends. Numbers for 6 recent weeks are 39, 44, 21, 26, 39, and 35.

  (a) State the mode (if one exists).

  (b) Find the median. **Show work/explanation.**

  (c) Determine the sample mean. **Show work**

  (d) Using the sample mean found in part (c), and given that the sample standard deviation of the data set above is 7.65, what percentage of the data set falls within one standard deviation of the mean? **Show work/explanation.**

(d) _____

A. 68.3%	C. 50.0%
B. 83.3%	D. 66.7%

24. If the probability distribution for the random variable X is given in the table, what is the expected value of X? **Show work.**

x_i	-30	10	20	60
p_i	0.40	0.30	0.20	0.10

25. A marketing survey of 2000 randomly-selected convenience store customers found that 1415 of them bought a glazed donut yesterday. 1605 said they bought a frosted crème-filled donut yesterday. 180 customers said they bought neither yesterday.

   (a) What is the probability that a single randomly-selected customer bought *only* a frosted crème-filled donut yesterday? *Round answer to nearest hundredth (two places after decimal)* **Show work.**

   (b) Let G = {customers who bought a glazed donut yesterday} and C = {customers who bought a frosted crème-filled donut yesterday}.   Determine the number of attendees belonging to each of the regions I, II, III, IV.
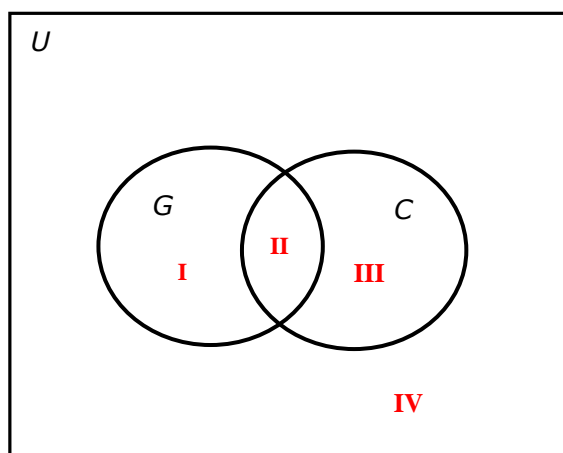
Region I: _____   Region II: _____   Region III: _____   Region IV: _____