

Introduction to Financial Mathematics – Semester 1, 2017

Assignment 7 Algebra

To be handed in by 2pm, Monday 8th of May

Examples:

- A. Let the technology (input-output) matrix for an open economy comprised of fishing, agriculture and mining industries be

$$A = \begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.4 \end{bmatrix}.$$

If the surpluses of 110 units of fishing output and 50 units of each of agriculture and mining goods are desired, find the output of each industry.

- B. Use your knowledge of closed Leontief economic models to complete the entries of the input-output matrix A and then find the output for the industries in the model.

$$A = \begin{array}{ccc|l} & G & I & H \\ \hline 0.4 & 0.2 & * & \textit{Government} \\ 0.2 & * & 0.3 & \textit{Industry} \\ * & 0.5 & 0.5 & \textit{Households} \end{array}$$

Questions

- The input-output matrix of a closed economic model is $A = \begin{bmatrix} 0 & 5/6 \\ 1 & 1/6 \end{bmatrix}$. Solve the closed Leontief economic model equation $(I - A)\mathbf{x} = \mathbf{0}$ for the production vector \mathbf{x} .
- In village of 90 people, each person either goes fishing or gathers nuts. All food is shared between the villagers and none is traded outside the village. The production of 3 units of fish requires two people and one unit of fish. The production of 1 unit of nuts requires one person. Each person in the village requires $1/3$ units of fish and $2/3$ units of nuts.
 - Write the input-output matrix A for the closed Leontief economic model which describes this village. **Hint:** the matrix is 3×3 and the sectors are fish, nuts and people (labour).
 - Solve the closed Leontief economic model equation $(I - A)\mathbf{x} = \mathbf{0}$ by Gauss–Jordan elimination to find the production of fish and nuts. **Hint:** There are 90 people in the village so set the people component of the production vector \mathbf{x} to 90.
 - In this village, how many people go fishing and how many people gather nuts?

3. Consider matrix $A = \begin{bmatrix} 0 & 2 & -2 \\ 3 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$.

- (a) Calculate $\det(A)$.
- (b) Find all cofactors of A and find A^{-1} .

4. Three neighbours have backyard vegetable gardens. Neighbour T grows tomatoes, neighbour C grows corn and neighbour L grows lettuce. The neighbours only sell their produce to each other. Neighbour T buys $1/5$ of the tomatoes, $1/5$ of the corn and $3/5$ of the lettuce. Neighbour C buys $2/5$ of the tomatoes, $1/5$ of the corn and $1/5$ of the lettuce.

- (a) What portion of each crop does neighbour L get?
- (b) What is the production vector \mathbf{x} ?
- (c) If the highest priced crop sells for \$42, what prices should be assigned to each of the crops?

5. The matrix

$$B = \begin{bmatrix} 0.2 & 0 & 0.2 & 0.3 & 0.1 \\ * & * & * & * & * \\ 0 & 0.4 & 0.5 & 0.5 & 0.5 \\ 0.3 & 0 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & 0.1 & 0.2 & 0 \end{bmatrix}$$

is the input-output matrix of a closed Leontief economic model.

- (a) Complete the missing row.
- (b) If industry four (that is, the industry described by the fourth column) produces 360 units, find the production vector \mathbf{x} . Give your final answers correct to one decimal place. (Hint: MATLAB might be helpful here.)