WED Apr. 26

Problem Set G: The Totient Function

This problem set concerns the properties of Euler's totient function φ..

1. Consider the case $p = 7$. For each of the numbers $a = 1, \ldots, 6$ and each exponent $n = 1, \ldots, 6$, find the remainder of a^n when divided by 7. Arrange this information in a 6x6 table, and describe any patterns that you find.

2. The Euler totient function $\varphi(n)$ is defined this way: among the numbers $1, \ldots, n$, $\varphi(n)$ is the number of numbers which have no common factor with n. Thus $\varphi(1) = 1$, $\varphi(2) = 1$, and $\varphi(4) = 2$. (Check these.) Compute $\varphi(n)$ for $n = 1, \ldots, 12$.

3. Prove that $\varphi(p) = p - 1$ if p is prime, and that $\varphi(p^n) = p^n - p^{n-1}$ for any positive integer n. (Hint: which of the numbers involved *do* have a common factor with p^n?)

4. Suppose that p and q are two distinct odd primes. Prove that $\varphi(pq) = \varphi(p)\varphi(q)$. (Hint: if a has a common factor with pq, what are the possibilities for that common factor?)