

Math 551 - Spring 2017 - Problem Set 4

Due Friday, March 31

1. Solve the spherical triangle $\triangle ABC$ if $\angle a = 60^\circ$, $\angle b = 70^\circ$, $\angle c = 80^\circ$.
(Note: to solve a spherical triangle means to find all 6 of the angles of the spherical triangle $\angle A, \angle B, \angle C, \angle a, \angle b, \angle c$.)
2. Assume that the earth is a perfect sphere of radius 3960 miles. By the *heading*, $h(\vec{a}, P)$, of a directed, great circle arc \vec{a} at the point P on the arc we mean the angle made with the “latitude” circle.

You are to “file a flight plan” for a direct flight from Syracuse to Cape Town, South Africa. Assume that the map coordinates of Syracuse are $43^\circ N, 76^\circ W$ and the map coordinates of Cape Town are $34^\circ S, 18^\circ E$. The flight plan should include the following items:

- (a) the conversion of the navigational coordinates of Syracuse and Cape Town to spherical coordinates - (ϕ_B, θ_B) for Syracuse and (ϕ_C, θ_C) for Cape Town;
- (b) a complete solution of the triangle with vertices: A (North Pole), B (Syracuse) and C Cape Town;
- (c) the direction of flight upon leaving Syracuse;
- (d) the length of your flight in miles and its duration in hours, assuming a constant speed of 700 mph;
- (e) Identify the when (flight time) and where (longitude) you will cross the equator.

3. Consider the triangle $\triangle ABC$ Where $\vec{A} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\vec{C} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$.
Find the coordinates of the vertices of the dual triangle $\triangle A^*B^*C^*$.

4. Let $\triangle ABC$ be a triangle on the unit sphere with $\angle A = \frac{\pi}{2}$, $\angle B = \frac{\pi}{3}$, $\angle C = \frac{\pi}{4}$.

- (a) Compute the arc length of a
- (b) This triangle may be used to tile the sphere; that is, the entire surface of the sphere may be cut up into triangles congruent to $\triangle ABC$. How many triangles will there be in such a tiling?