(1 point) Find the power series representation for

$$f(x) = \int_0^x \frac{\tan^{-1} t}{t}\, dt.$$

$$f(x) = \sum_{n=1}^{\infty} (-1)^{e_n} a_n x^{p_n},$$

where $e_n =$

○ **A.** n ○ **B.** n - 1 ○ **C.** 0

and $a_n =$ ______ ,

and $p_n =$ ______ .

(1 point) Find the sum of

$\sum_{n=1}^{\infty} n(n+1)x^n =$ ______

for ______ $< x <$ ______ .

(1 point) Find the first four nonzero terms of the Taylor series about 0 for the function $f(x) = \sqrt{1+x}\cos(6x)$. Note that you may want to find these in a manner other than by direct differentiation of the function.

$\sqrt{1+x}\cos(6x) =$ ______ + ______ + ______ + ______ $+ \cdots$

(1 point) Find the Taylor series about 0 for each of the functions below. Give the first three non-zero terms for each.

A. $\frac{1}{\sqrt{1+x^4}} =$ ______ + ______ + ______ $+ \cdots$

B. $2\cos(x) + x^2 =$ ______ + ______ + ______ $+ \cdots$

For each of these series, also be sure that you can find the general term in the series!

(1 point) The function $f(x) = 7x^2 \arctan(x^3)$ is represented as a power series

$f(x) = \sum_{n=0}^{\infty} c_n x^n$.

What is the lowest term with a nonzero coefficient.

$n =$

Find the radius of convergence R of the series.

$R =$.

(1 point)

Use a Maclaurin series derived in the text to derive the Maclaurin series for the function $f(x) = \int \frac{\sin(x)}{x}\, dx, f(0) = 0$. Find the first 4 nonzero terms in the series, that is write down the Taylor polynomial with 4 nonzero terms.

(1 point)

Find the sum of the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{9n}}{n!}$$

It will be a function of the variable x.

(1 point) The function $f(x) = \ln(1 - x^2)$ is represented as a power series

$f(x) = \sum_{n=0}^{\infty} c_n x^n$.

Find the FOLLOWING coefficients in the power series.

$c_0 =$

$c_1 =$

$c_2 =$

$c_3 =$

$c_4 =$

Find the radius of convergence R of the series.

$R =$.

(1 point) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$$

Answer:

Note: Give your answer in interval notation

(1 point) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(6x-4)^n}{n^2}$$

Answer: [ ]

Note: Give your answer in interval notation

(1 point) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n+2}}$$

Answer: [ ]

Note: Give your answer in interval notation.

(1 point) The function $f(x) = \frac{4}{(1+4x)^2}$ is represented as a power series:

$$f(x) = \sum_{n=0}^{\infty} c_n x^n$$

Find the first few coefficients in the power series.

$c_0 =$ [ ]

$c_1 =$ [ ]

$c_2 =$ [ ]

$c_3 =$ [ ]

$c_4 =$ [ ]

Find the radius of convergence R of the series.

$R =$ [ ] .

(1 point) Find the convergence set of the given power series:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2}$$

The above series converges for [ ] $\leq x \leq$ [ ] .

Enter "infinity" for ∞ and "-infinity" for $-\infty$.

(1 point) A famous sequence f_n, called the Fibonacci Sequence after Leonardo Fibonacci, who introduced it around A.D. 1200, is defined by the recursion formula

$$f_1 = f_2 = 1, \quad f_{n+2} = f_{n+1} + f_n.$$

Find the radius of convergence of

$$\sum_{n=1}^{\infty} f_n x^n.$$

Radius of convergence: ______ . Hint: The limit of f_{n+1}/f_{n} is (1+sqrt{5})/2.

(1 point) Find Taylor series of function $f(x) = \ln(x)$ at $a = 4$.

$$(f(x) = \sum_{n=0}^{\infty} c_n (x-4)^n)$$

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the interval of convergence.

The series is convergent:

from $x =$ ______ , left end included (Y,N): ______

to $x =$ ______ , right end included (Y,N): ______

(1 point) Find the first five non-zero terms of Taylor series centered at $x = 5$ for the function below.

$$f(x) = \sqrt{10x - x^2}$$

Answer: $f(x) =$ ______ + ______ + ______ + ______ + ______ $+ \cdots$

What is the radius of convergence?

Answer: $R =$ ______

(1 point) Consider the power series

$$\sum_{n=1}^{\infty} \frac{n^6 (x-8)^n}{4 \cdot 8 \cdot 12 \cdot \cdots \cdot (4n)}.$$

Find the radius of convergence R. If it is infinite, type "infinity" or "inf".

Answer: $R =$ ______

What is the interval of convergence?

Answer (in interval notation): ______

(1 point) Find the first five non-zero terms of power series representation centered at $x = 0$ for the function below.

$$f(x) = \frac{3x^2}{(x-2)^2}$$

Answer: $f(x) =$ ____ + ____ + ____ + ____ + ____ $+ \cdots$

What is the radius of convergence?

Answer: $R =$ ____

(1 point) Find the first five non-zero terms of power series representation centered at $x = 0$ for the function below.

$$f(x) = \frac{x^3}{1+3x}$$

Answer: $f(x) =$ ____ + ____ + ____ + ____ + ____ $+ \cdots$

What is the interval of convergence?

Answer (in interval notation): ____

(1 point) Use the ratio test to find the radius of convergence of the power series

$$4x + 16x^2 + 64x^3 + 256x^4 + 1024x^5 + \cdots$$

$R =$ ____

*(If the radius is infinite, enter **Inf** for R.)*

(1 point) Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{n^3 x^{4n}}{2^{4n}}.$$

interval of convergence = ____

*(Enter your answer as an interval: thus, if the interval of convergence were $-3 < x \leq 5$, you would enter **(-3,5]**. Use **Inf** for any endpoint at infinity.)*