Propositional Equivalences and Quantifiers

1. Show that each of the following equivalences is a tautology in two ways: (1) using a truth table, and (2) using logical equivalences(see list on last page). List which equivalences you are using for each line in part (2).

"p" "q"

(a) $(p \wedge q) \rightarrow p$

p	q	p	$p \wedge q \rightarrow p$
T	T		T
T	F		
F	T		
F	F		

$\equiv \neg(p \wedge q) \vee p$ (Logical #### ·)

$\equiv (\neg p \vee \neg q) \vee p$ (De Morgans)

$\equiv (\neg p \vee p) \vee \neg q$ (assoc/comm)

$\equiv T \vee \neg q$ (Negation)

$\equiv T$ (domination)

"p" → "q"

(b) $\neg(p \rightarrow q) \rightarrow \neg q$

$\equiv (p \wedge \neg q) \rightarrow \neg q$ (Logical dom)

$\equiv (p \wedge \neg q) \vee \neg q$ (logical do ne)

$\equiv (p \wedge \neg q) \vee \neg q$ (DeMorgans)

$\equiv (\neg p \vee q) \vee \neg q$ (assoc/comm)

$\equiv (\neg q \vee q) \vee \neg p$ (double Negation)

$\equiv (T) \vee \neg p$

$\equiv$

"p" "q"

(c) $(p \wedge q) \rightarrow (p \rightarrow q)$

$\equiv \neg(p \wedge q) \rightarrow (p \rightarrow q)$ (Logical Equivalence)

$\equiv (\neg p \vee \neg$ $)$

2. Translate the following statement into English. Let $C(x)$ denote the propositional function "x is a comedian" and $F(x)$ denote "x is funny", where the domain consists of all people.

$$\forall x(C(x) \wedge F(x)).$$

Is this a true claim?

Translate $\exists x(C(x) \rightarrow F(x))$. Is this true or false?

What is the distinction between $\exists x(C(x) \rightarrow F(x))$ and $\exists x(C(x) \wedge F(x))$?

Useful Equivalences

$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's Laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation Laws
1 $p \to q \equiv \neg p \vee q$ $\neg(p \to q) \equiv p \wedge \neg q$ (proved in class)	Logical Equivalences Involving Conditionals