The Lotka-Volterra model for population growth can describe symbiotic relationships, parasitic relationships, and predator-prey relationships. This pair of differential equations is from the last category.

Coyotes prey on jackrabbits; without the jackrabbits, the coyote population won't survive.

Jackrabbits experience exponential growth that is checked by the size of the coyote population. The more of each species that there are occupying this environment, the more likely it is that there will be an encounter between two individuals which usually ends badly for the jackrabbit.

$$\begin{cases} \frac{dJ}{dt} = .08J - .001JC & J(0) = 300 \\ \frac{dC}{dt} = -.02C + .00002JC & C(0) = 50 \end{cases}$$

Before worrying about MATLAB, try to answer a few questions:

1. Can $\frac{dJ}{dt}$ or $\frac{dC}{dt}$ ever be zero? What would that mean in terms of the populations of rabbits and coyotes?

2. At the point we're starting at, $J(0) = 300$ and $C(0) = 50$, will the populations be increasing or decreasing?

3. Can you find positive values of J and C that make:

   (a) $\frac{dJ}{dt}$ and $\frac{dC}{dt}$ both positive?
   (b) $\frac{dJ}{dt}$ and $\frac{dC}{dt}$ both negative?
   (c) $\frac{dJ}{dt}$ positive and $\frac{dC}{dt}$ negative?
   (d) $\frac{dJ}{dt}$ negative and $\frac{dC}{dt}$ positive?

   There should be four different phases of growth or decay for these populations. How long do they each last?

What I'd like to see from you:

1. Using Euler's method, try to forecast the future of these populations, barring any unforeseen changes in climate and the behavior of other species. Graphs of J vs. t, C vs. t, and J vs. C would all be helpful; go out several decades (maybe even a century).

2. Suppose there is a disaster; a wildfire, a volcano, a meteor; there's no need to be specific. It decimates the population of the jackrabbits and also effects the coyotes, but not as drastically. During the month when the disaster hits, the jackrabbit population suddenly drops to 10% of its size the previous month, and the coyote population drops to 40% of its size the previous month.

   (a) If the disaster happens during a time when both populations are growing, will the two species survive?
   (b) If the disaster happens during a time when both populations are declining, will the two species survive?