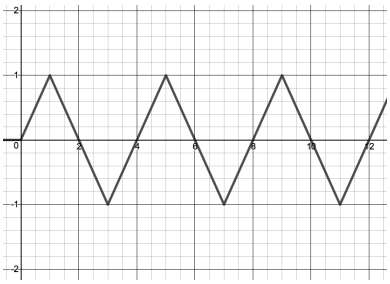
Math 2A Sawtooth-Wave Driving Oscillator
Morriss

Consider a certain mechanical system of a 80-kg mass attached to a 180N/m spring and a damper with resistance force 1N at 1m/sec.

The system is initially at rest at equilibrium, and driven by an outside force in the form of a sawtooth sine-wave oscillator with amplitude 1 and period 4 seconds, operating for one minute on a one-minute delay.



The system can be modeled by the IVP
$80y'' + y' + 180y = g(t)$, $y(0) = y'(0) = 0$;
where $g(t) = [u(t - 60) - u(t - 120)] \cdot f(t)$,
and f(t) is the sawtooth wave graphed here.

Do what you can with this. In particular,

a) Discuss the mechanical system, predict the system response, and sketch a rough solution graph.
b) Express f(t) in terms of Heaviside step functions
c) Find the Laplace transforms of f(t), g(t), and y(t)
d) Invert the transform of y(t) to find an analytic solution
e) Produce a graph of the first ten minutes of system response
f) Ask and answer some interesting questions about the solution.

If you can, explore the effect of leaving the oscillator on for a longer time: 2 minutes, 5, 10, etc. Try to explain what you see.

I'll take what you come up with on Tues Nov 29 at 7:30AM