Chapter ~~17~~ 16 Exam Name: ________________

1. Evaluate the given line integral. You may use any method you deem effective.

   (a) (8 points) $\int_C y\ dx - x\ dy$ where C is the line segment from $(2, 4)$ to $(4, 7)$.

   (b) (8 points) $\int_C x^2 y\ ds$ where C is the top half of the circle $x^2 + y^2 = 4$.

   (c) (14 points) $\int_C e^{-y}\ dx - xe^{-y}\ dy$ where C is any simple curve from $(1, 0)$ to $(0, 2)$.

(d) (14 points) $\oint_C \hat{\mathbf{F}} \cdot d\,\hat{\mathbf{r}}$ where $\hat{\mathbf{F}} = e^x \sin(y)\,\hat{\mathbf{i}} + e^x \cos(y)\,\hat{\mathbf{j}}$ where C is the positive orientation of the circle $x^2 + y^2 = 25$.

2. (12 points) Prove $\text{div}(\text{curl}(\hat{\mathbf{F}})) = 0$ for any vector field $\hat{\mathbf{F}}$.

3. (8 points) Determine if the vector field $\hat{\mathbf{F}} = (e^x + y^2 e^{xy})\ \hat{\mathbf{i}} + (1+xy)e^{xy}\ \hat{\mathbf{j}}$ is conservative. If it is, find a function $f(x, y)$ such that $\nabla f = \ \hat{\mathbf{F}}$.

4. Let $\hat{\mathbf{F}} = xy^3\ \hat{\mathbf{i}} + \left(\frac{3}{2}x^2y^2 + y\right)\ \hat{\mathbf{j}}$.

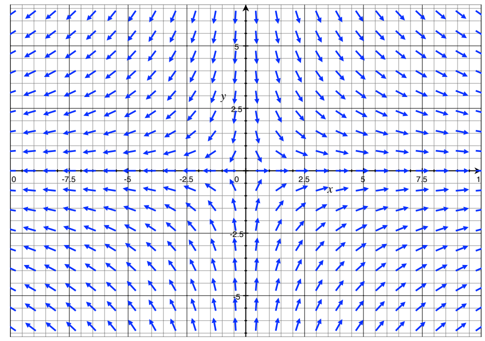
   (a) (8 points) Determine if $\hat{\mathbf{F}}$ is conservative, and if so find a function $f(x, y)$ such that $\nabla f = \ \hat{\mathbf{F}}$.

   (b) (6 points) Use your answer from part a) to evaluate $\int_C \hat{\mathbf{F}} \cdot d\,\hat{\mathbf{r}}$ where C is the curve defined by $\hat{\mathbf{r}}(t) = \langle 3\cos(\frac{\pi t}{2}), \frac{t^3}{2}\rangle, 0 \leq t \leq 2$.

   (c) (4 points) Use your answer from part a) to quickly evaluate $\oint_C \hat{\mathbf{F}} \cdot d\,\hat{\mathbf{r}}$ over any simple closed curve C.

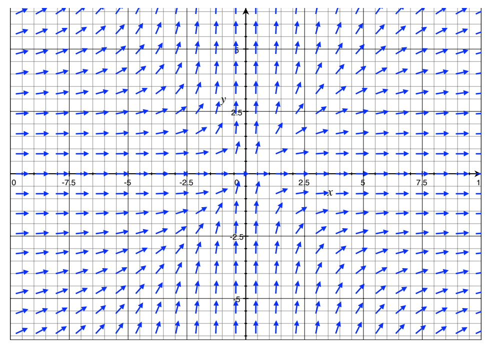
5. (2 points each) Match each function $f(x, y)$ with the plot of the gradient field ∇f.

A. $f(x, y) = x^2 - y^2$



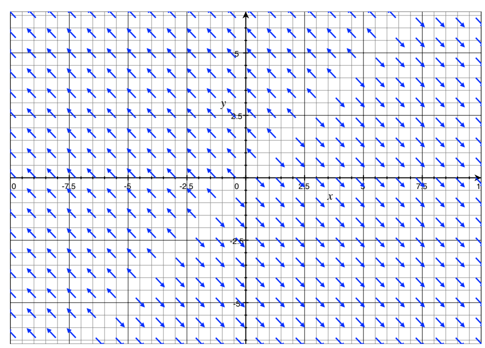
I.

B. $f(x, y) = (x - y)^2$



II.

C. $f(x, y) = \frac{x^3 + y^3}{3}$



III.

6. (12 points) Find the divergence and curl of the vector field $\hat{\mathbf{F}} = \ln(xyz)\ \hat{\mathbf{i}} + \ln(xy)\ \hat{\mathbf{j}} + \ln(yz)\ \hat{\mathbf{k}}$. Determine if $\hat{\mathbf{F}}$ is conservative, incompressible, both or neither.