Suppose we want to design a ride for the AZ state fair. Have you heard of a ride named the "Reverse Bungee", or sometimes referred to as a "Catapult Bungee"? From Wikipedia:

"The ride consists of two telescopic gantry towers mounted on a platform, feeding two elastic ropes down to a two person passenger car constructed from an open sphere of tubular steel. The passenger car is secured to the platform with an electro-magnetic latch as the elastic ropes are stretched. When the electromagnet is turned off, the passenger car is catapulted vertically with a g-force of 3 to 5, reaching an altitude of between 50 and 80 meters (180 to 260 ft).

The passenger sphere is free to rotate between the two ropes, giving the riders a chaotic and disorienting ride. After several bounces, the ropes are relaxed and the passengers are lowered back to the launch position."

Let's try to design something using our model for springs.

$$mx^{''} + \beta x^{'} + kx = 0$$

This ride wouldn't allow the riders to tumble around quite as chaotically; it would just shake them up a bit.

Imagine a gondola attached to a spring. The spring goes from the top of the gondola upward to an anchor that is at an as-of-yet undetermined point above the ground. (This is different from what we described in class, where the spring was anchored to the bottom.)

People are supposed to enter the gondola while it is pinned to the ground, be strapped in, and then enjoy a bouncy ride as the gondola is released and the spring recoils. Clamps on the back of the gondola keep it running along a guide pillar next to the ride, so the motion is straight up and down (rectilinear motion).

For now let's assume:

1. The combined weight of the people and the gondola is 1000 lbs. (Don't forget to change this into mass! Mass times g would be weight.)

2. The initial position is 75 feet below the equilibrium point of the spring and 1000 lbs gondola-passenger payload. ($x(0) = 75$) The anchor for the spring is much higher than that, and hopefully won't have to be considered.

3. The initial velocity of the gondola is 0. ($x^{'}(0) = 0$)

Other than that, we have control over most of the other variables by using different springs (with different values of k), and controlling how tightly the clamps grip the guiding pillar (influencing β, the drag constant).

Try to find values for β and k that are positive and would result in a ride that meets the following criteria:

The system should be underdamped. An overdamped ride would not be very bouncy or exciting.

Oscillations greater than 10 feet above or below equilibrium should continue occurring for at least 30 seconds.

There should be at least 6 times before those 30 seconds where the gondola reaches a turning point (3 minimums, 3 maximums).

The acceleration ($x''(t)$) should stay less than $5g$.

In a written document, please provide:

1. The values you used for β and k and provide a graph of the position of a typical ride for a 1000 lbs payload.
2. Include some convincing evidence that the riders won't experience more than $5g$ of force.
3. For safety reasons, also provide graphs and a small analysis of what would happen for an unusually small payload (700 lbs) or an unusually large one (1300 lbs). Determine the maximum altitude reached and estimate the time it takes for the oscillations to shrink to less than 10 feet (at which point we stop everything by reactivating our anchor and dragging the gondola back to earth).

In class we might discuss script files and the `diff` command, which might help with some of this.