$\qquad$
$\qquad$

## 4-3 Enrichment

## Squaring the Circle

Squaring a circle means to construct a square that has the same area as a given circle. The area of the unit circle is exactly $\pi$ square units. To construct a square with this exact area, each side would have to have a length of $\sqrt{\pi}$ units. It is impossible to construct a segment with a length $\sqrt{\pi}$ using a straight edge and compass, so it is impossible to construct a square with an exact area of $\pi$ square units.

This problem can be solved by rolling a unit circle a half turn along a straight line. This gives a line that has an exact length of $\pi$ units.

In the diagram, the segment $A B$ has the exact length $\pi$ units because the unit circle was unrolled a half
 turn. When a semicircle is drawn from $A$ to $C$, it follows that diameter $A C=\pi+1$. Extend the vertical radius down until it intersects arc $A C$ at point $D$.

Since $B D$ is the geometric mean between $A B$ and $B C, A B \times B C=B D^{2}$.
Since $A B=\pi$ and $B C=1, B D=\sqrt{\pi}$.

## Exercises

1. Use a circular object, such as a coin, to perform the above construction.

Then find the radius of the object and calculate the area of the object. Measure the side of the square and calculate the area of the square. How close were the areas? What might account for the differences in the areas?
2. Pick a different object and repeat the process in Exercise 1.
3. What is the ratio of the side length of the square to the radius of the circle from Exercise 1? Exercise 2? What is the significance of this ratio?
$\qquad$
$\qquad$
$\qquad$

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Let $(-3,-\sqrt{3})$ be a point on the terminal side of an angle $\theta$ in standard position.
a. Give the measures of three angles that fit the description.
b. Explain how to find the cosine of these angles. Give the cosine of these angles.
2. Write the coordinates of a point in Quadrant II. Find the values of the six trigonometric functions of an angle in standard position with this point on its terminal side.
3. A playground is being built next to a circular fountain in a park. A fence will be installed around the playground for safety, as shown below.

a. How long will the fence need to be to enclose the entire playground?
b. The park commission is planning to enlarge the playground. Do you think it should be enlarged to the east or to the west? Explain your reasoning.
4. Given $y=5 \sin \left(2 x+\frac{\pi}{2}\right)$, find the amplitude, period, and phase shift. Explain the significance of each.
$\qquad$
$\qquad$
$\qquad$

## 4-7 Enrichment

## Triangle Challenge

A surveyor took the following measurements from two irregularly shaped pieces of land. Some of the lengths and angle measures are missing. Find all missing lengths and angle measures. Round lengths to the nearest tenth and angle measures to the nearest minute.
1.

2.


