# ECONOMICS 305: Intermediate Macroeconomics

# Spring 2016 ASSIGNMENT #5

**DUE DATE: Thursday, April 21, noon.**

## Question 1. The Solow Model of Economic Growth

Consider the following Solow growth model with technological change and population growth:

$$Y_t = K_t^{0.5}(A_t N_t)^{0.5} \quad (1)$$

$$S_t = sY_t, 0 < s < 1 \quad (2)$$

$$K_{t+1} - K_t = I_t - \delta K_t \quad (3)$$

$$\frac{N_{t+1}}{N_t} = 1 + g_N, g_N = 0.01 \quad (4)$$

$$\frac{A_{t+1}}{A_t} = 1 + g_A, g_A = 0.02 \quad (5)$$

a) Explain in words what each of these equations means or describes.

b) Write down the goods market equilibrium condition for the model.

c) Combine the goods market equilibrium condition with equations (1) through (3) to find an equation that describes the change in the capital stock between dates *t* and *t+1* in terms of the levels of inputs to production at date *t*. Explain in words what determines this change over time; whether it is positive or negative or zero.

d) Now take each variable in the model and divide it by $A_t N_t$. Use these transformed variables to re-express the equation you derived in c) as an equation that describes the change in the capital stock per effective worker between dates *t* and *t+1*. Explain in words what determines this change over time; whether it is positive or negative or zero.

e) Define and describe in words a long-run, steady state equilibrium of this economy. Depict a long-run, steady state equilibrium in a diagram and label the diagram carefully. What condition on the equation that you derived in d) would measure or captured this steady state?

f) In the steady state equilibrium, what will be the numerical values of the growth rates of aggregate output, the aggregate capital stock, aggregate investment, and aggregate savings? What will be the numerical values of the growth rate of output per worker,

and capital per worker? What will be the numerical values of the growth rate of output per effective labor unit and capital per effective labor unit?

g) What would be the qualitative impact of an increase in *s* for the steady state level of capital per effective worker and output per effective worker? Show this in a diagram.

h) What would be the qualitative impact of an increase in *s* for the steady state growth rates of output, capital, savings and investment? What is the impact of an increase in *s* for output per worker?

i) What factors will cause a change the steady state growth rate of this economy? What types of policies would a government have to enact to increase the steady state growth rate of the economy? Would an increase in the steady state growth rate of the economy increase living standards in the steady state? Explain carefully.

**Question 2. More on the Solow Growth Model**

a) Re-do Question 1, parts b) through d), assuming that the production function is given by

$$Y_t = K_t^{0.3}(A_t N_t)^{0.7} \qquad (1')$$

b) If the production function is given by (1') how does this affect your answers to Question 1 parts e) through i), if at all?

c) Derive expressions for the steady state level of capital per effective worker and output per effective worker for the case where the production function is given by (1) and the case where it is given by (1').

d) Assume that depreciation is 10 percent per year, and the savings rate is twenty percent. What are the steady state levels of capital and output per effective worker for the case where the production function is given by (1) and the case where it is given by (1'). Compare them and explain in words how and why they are different, if they are.

e) Now assume that the savings rate increases from twenty percent to thirty percent. How does this affect the steady state levels of capital and output per effective worker when the production function is given by (1) and when it is given by (1'). Compare them and explain in words how and why they are different, if they are.

f) Now assume that the growth rate of technological change increases from 2 percent to 3 percent per annum. How does this affect the steady state levels of capital and output

per effective worker when the production function is given by (1) and when it is given by (1'). Compare them and explain in words how and why they are different, if they are.

**Question 3. Technology in the Short-Run and Medium-Run**

Assume that the economy's labor market features firms who produce goods according to the production function

$$Y = AN \tag{1}$$

and set price at a markup over marginal cost of production

$$P = \frac{(1+m)W}{A} \tag{2}$$

Workers negotiate wages in bargaining with firms, such that wages are given by

$$W = P^e A^e F(u, z) \tag{3}$$

a) Explain in words what equations (1) through (3) represent. What is the marginal cost of production in this model and why?

b) For now, assume that $A = A^e = 1$. Combine equations (2) and (3) to derive the aggregate supply relation, and plot it as a curve in a diagram with aggregate output on the horizontal axis and the price level on the vertical axis. Explain carefully why the aggregate supply curve that you have plotted has the slope that it does.

c) Let the aggregate demand relation be described by the equation

$$Y = Y\left(\frac{M}{P}, G, T\right).$$

Plot this aggregate demand relation as a curve in a diagram with aggregate output on the horizontal axis and the price level on the vertical axis. Explain carefully why the aggregate demand curve that you have plotted has the slope that it does.

d) Depict in a separate diagram a medium run equilibrium for the macro-economy, using aggregate supply and aggregate demand curves. What is the relationship of the medium run equilibrium output level depicted in this diagram to the natural rate of unemployment?

e) Now suppose that the economy starts in the medium run equilibrium that you have depicted in d) and that labor productivity $A$ unexpectedly doubles from 1 to 2. Assume that any effects, positive and negative, for aggregate demand cancel each other out

and that only the aggregate supply relation is affected. What effect does this have on the natural rate of unemployment? Show this is a diagram and explain your answer. What effect does it have in the *short run* for (i) unemployment, (ii) equilibrium output, and (iii) the price level? Show these short-run effects in a diagram and explain them carefully in words.