

Insurance Market and Adverse Selection

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Why Study Market for Insurance?

Table A. Average annual expenditures and characteristics of all consumer units and percent changes, 2012-14

| Item | 2012 | 2013 | 2014 | Percent change | |
|---------------------------------|----------|----------|----------|----------------|-----------|
| | | | | 2012-2013 | 2013-2014 |
| Average income before taxes | \$65,596 | \$63,784 | \$66,877 | -2.8 | 4.8 |
| Average annual expenditures | \$51,442 | \$51,100 | \$53,495 | -0.7 | 4.7 |
| Food | 6,599 | 6,602 | 6,759 | 0.0 | 2.4 |
| Food at home | 3,921 | 3,977 | 3,971 | 1.4 | -0.2 |
| Food away from home | 2,678 | 2,625 | 2,787 | -2.0 | 6.2 |
| Housing | 16,887 | 17,148 | 17,798 | 1.5 | 3.8 |
| Shelter | 9,891 | 10,080 | 10,491 | 1.9 | 4.1 |
| Owned dwellings | 6,056 | 6,108 | 6,149 | 0.9 | 0.7 |
| Rented dwellings | 3,186 | 3,324 | 3,631 | 4.3 | 9.2 |
| Apparel and services | 1,736 | 1,604 | 1,786 | -7.6 | 11.3 |
| Transportation | 8,998 | 9,004 | 9,073 | 0.1 | 0.8 |
| Gasoline and motor oil | 2,756 | 2,611 | 2,468 | -5.3 | -5.5 |
| Vehicle insurance | 1,018 | 1,013 | 1,112 | -0.5 | 9.8 |
| Healthcare | 3,556 | 3,631 | 4,290 | 2.1 | n/a |
| Health insurance | 2,061 | 2,229 | 2,868 | 8.2 | n/a |
| Entertainment | 2,605 | 2,482 | 2,728 | -4.7 | 9.9 |
| Cash contributions | 1,913 | 1,854 | 1,788 | -4.1 | -2.5 |
| Personal insurance and pensions | 5,591 | 5,528 | 5,726 | -1.1 | 3.6 |
| All other expenditures | 3,557 | 3,267 | 3,548 | -8.2 | 8.6 |

n/a - Because of the questionnaire change for health insurance, the 2013-14 percent change is not strictly comparable to prior years.

Spending patterns, 2013-14

- ▶ Spending on insurance accounts for no small part of household's expenditures
- ▶ As of 2014, 10.7% of the average household expenditure in the U.S. is spent on personal insurance payments and pension contribution

Why Study Market for Insurance?

Average household expenditure, by province
(British Columbia)

| | 2013 | 2014 |
|---|---------------|---------------|
| | \$ | |
| Total expenditures | 78,851 | 80,776 |
| Total current consumption | 61,379 | 60,931 |
| Food expenditures | 8,118 | 8,218 |
| Shelter | 18,889 | 18,497 |
| Household operation | 4,367 | 4,524 |
| Household furnishings and equipment | 2,075 | 1,987 |
| Clothing and accessories | 3,494 | 3,101 |
| Transportation | 11,298 | 11,511 |
| Health care | 2,775 | 2,522 |
| Personal care | 1,229 | 1,183 |
| Recreation | 3,960 | 4,180 |
| Education | 1,026 | 2,011 |
| Reading materials and other printed matter | 193 | 163 |
| Tobacco products and alcoholic beverages | 1,271 | 1,103 |
| Games of chance | 230 | 169 |
| Miscellaneous expenditures | 1,654 | 1,762 |
| Income taxes | 11,294 | 13,005 |
| Personal insurance payments and pension contributions | 4,210 | 4,263 |
| Gifts of money, alimony and contributions to charity | 1,969 | 2,576 |

Sources: Statistics Canada, CANSIM, table 203-0021 and Catalogue no. 62F0026M.
Last modified: 2016-02-12.

- ▶ As of 2014, 5.2% of the average household expenditure is spent on personal insurance payments and pension contribution
- ▶ In some countries like Canada, most people do not even have options to opt out of insurance
- ▶ Why???? And more...

Review of Consumer's Problem

- ▶ For reasons that will become clear soon, let's review basics of consumer's problem
- ▶ Suppose that there are n consumption goods x_1, x_2, \dots, x_n
- ▶ p_k : market price of good x_k
- ▶ Your income is $\$I$
- ▶ Your budget set is

$$\sum_{k=1}^n p_k x_k \leq I$$

- ▶ Consumer's problem: how much of each consumption good to consume?

Review of Consumer's Problem

- ▶ Your utility function is $u(x_1, x_2, \dots, x_n)$
- ▶ Often we assume that $u(x_1, x_2, \dots, x_n)$ is strictly quasiconcave
- ▶ This means that given two bundles of goods $a = (a_1, \dots, a_n)$ and $b = (b_1, \dots, b_n)$, if

$$u(a) \geq u(b)$$

then you prefer a "MIX" c of the bundles a and b to b

- ▶ A "mix" of two bundles of goods a and b , or a "*convex combination*" of two bundles a and b is

$$\begin{aligned} & \alpha a + (1 - \alpha)b \\ &= (\alpha a_1 + (1 - \alpha)b_1, \dots, \alpha a_n + (1 - \alpha)b_n) \end{aligned}$$

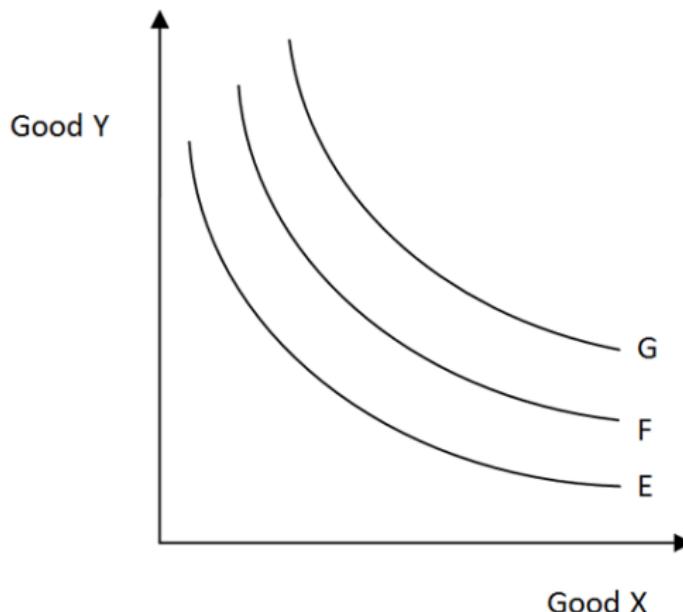
where α is some number between 0 and 1

- ▶ Then strict quasiconcavity implies

$$u(\alpha a_1 + (1 - \alpha)b_1, \dots, \alpha a_n + (1 - \alpha)b_n) > u(b_1, \dots, b_n)$$

Two-Good Case

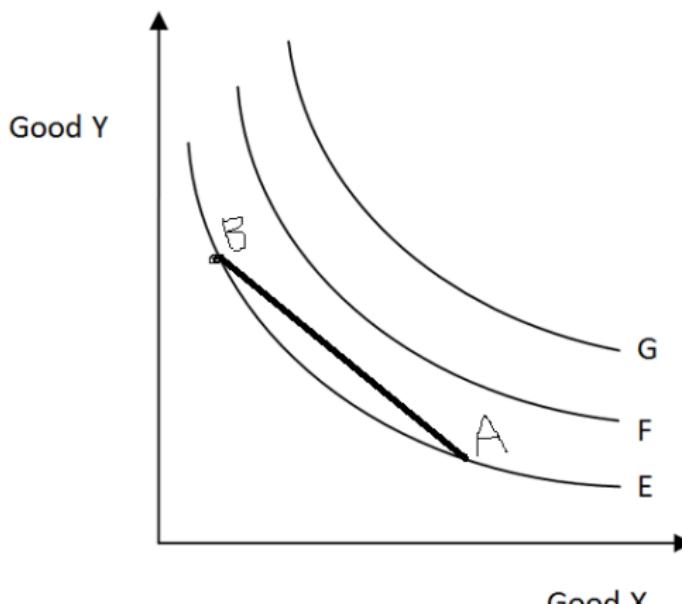
- ▶ There are two goods
- ▶ A strictly quasiconcave utility function $u(x, y)$ has indifference curves that look like



Two-Good Case

- With strict quasiconcavity, given two bundles $a = (a_1, a_2)$ and $b = (b_1, b_2)$ where $u(a) \geq u(b)$, you always prefer a mix of A and B

$$u(\alpha a_1 + (1 - \alpha)b_1, \alpha a_2 + (1 - \alpha)b_2) > u(b_1, b_2)$$



Two-Good Case

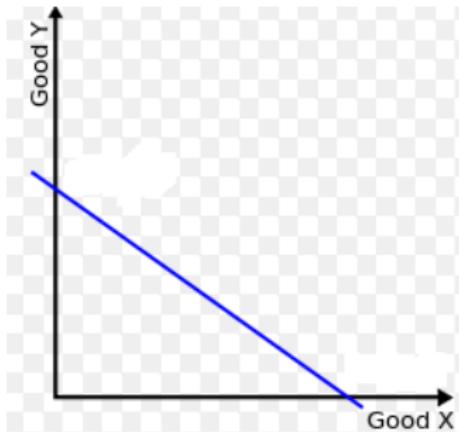
- ▶ If we assume that the utility function $u(x, y)$ are differentiable (i.e., we can take first derivative of it), then strict quasiconcavity implies that the marginal rate of substitution between X and Y is decreasing
- ▶ MRS at some particular consumption bundle (x_0, y_0) is

$$MRS(x_0, y_0) = \frac{\frac{\partial u(x_0, y_0)}{\partial X}}{\frac{\partial u(x_0, y_0)}{\partial Y}} = \frac{\text{Marginal utility of X at } (x_0, y_0)}{\text{Marginal utility of Y at } (x_0, y_0)}$$

- ▶ The interpretation of $MRS(x_0, y_0)$ is the value of good X TO YOU in terms of good Y when you consume the bundle (x_0, y_0)
- ▶ For example, if $MRS(x_0, y_0) = 5$, then when you consume the bundle (x_0, y_0) , the value of good X to you is five times that of good Y to you
- ▶ Along any indifference curve of a strictly quasiconcave utility function, MRS decreases as X increases, meaning that as you consume more and more of X , it is worth less and less to you in terms of good Y

Budget Set in Two Good Cases

- The budget set in the two-good case is $\{(x, y) : p_x x + p_y y \leq I\}$

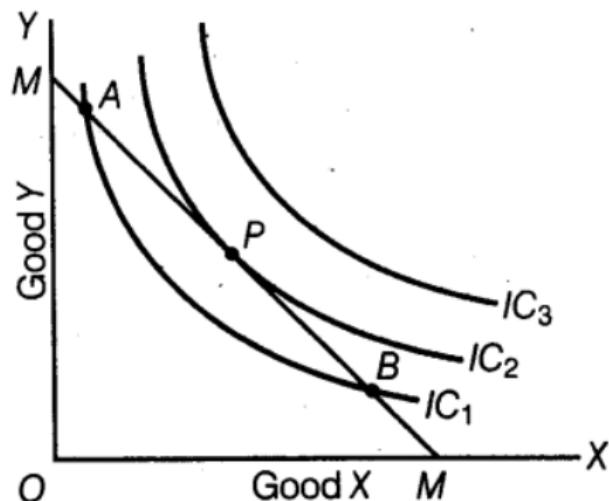


- What is the slope of the budget line?

$$-\frac{p_x}{p_y} = -\frac{\text{Market price of good X}}{\text{Market price of good Y}} = -\text{Market value of good X in terms of good Y}$$

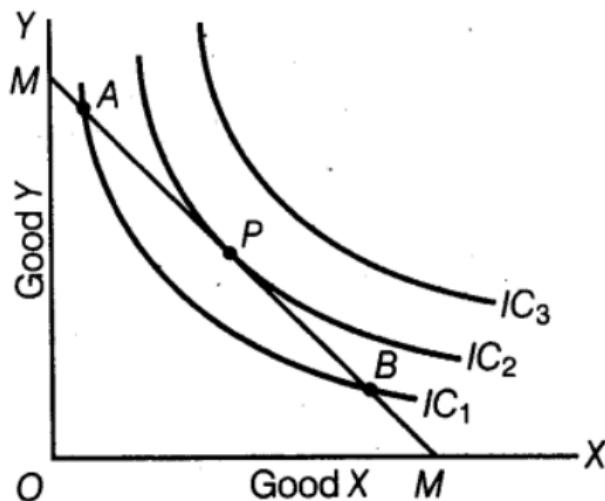
- For example, if $p_x = \$5$ and $p_y = \$1$, then the value of good X in the market is five units of good Y

Optimal Consumption Bundle



- ▶ At optimal bundle, the marginal rate of substitution =
$$\frac{\text{Price of good X}}{\text{Price of good Y}}$$

Optimal Consumption Bundle



- ▶ Why are A not optimal?
- ▶ At A, marginal rate of substitution is larger than the price ratio, or the value of X in terms of good Y to you is larger than the market value of good X in terms of good Y

Optimal Consumption Bundle

- ▶ This means that if you sell some of your good Y to buy good X in the market, your utility would rise
- ▶ For example, if at A, $MRS=5$ and $\frac{p_x}{p_y} = 4$, then you can sell 1 unit of Y to get $\frac{1}{4}$ units of X in the market
- ▶ However, because your value of X = $5 \times$ your value of Y at A, you gain more utility

$$\begin{aligned}& \text{-value of a unit of Y} + \frac{1}{4} \text{value of a unit of X} \\&= -\text{value of a unit of Y} + \frac{5}{4} \text{value of unit of Y} \\&= \frac{1}{4} \text{value of a unit of Y}\end{aligned}$$

- ▶ P is optimal because your value of X is the same as the market value of X; so there is no trade you can make with the market to increase your utility

Demand for Insurance

- ▶ Let's consider a typical situation that faces the consumer of insurance
- ▶ Suppose your initial wealth is w
- ▶ In the next time period, you know that
 - ▶ Some bad events occur and you will lose $\$L$ with probability p
 - ▶ Nothing happens and no change in your wealth with probability $1 - p$
- ▶ $v(x)$: your Bernoulli utility function
- ▶ Then your expected utility is...

$$pv(w - L) + (1 - p)v(w)$$

- ▶ That is, this is your expected from the lottery $(w - L, p; w, (1 - p))$

Demand for Insurance

- ▶ Then Sam comes along and makes an offer to you
 - ▶ "If you pay me q dollars now, I will pay you a dollar in the next period only if your wealth decreases"
 - ▶ "But if nothing happens, you do not get anything from me"
- ▶ In effect, Sam is offering insurance to you
- ▶ Then how will you determine your optimal consumption of insurance?

Demand for Insurance

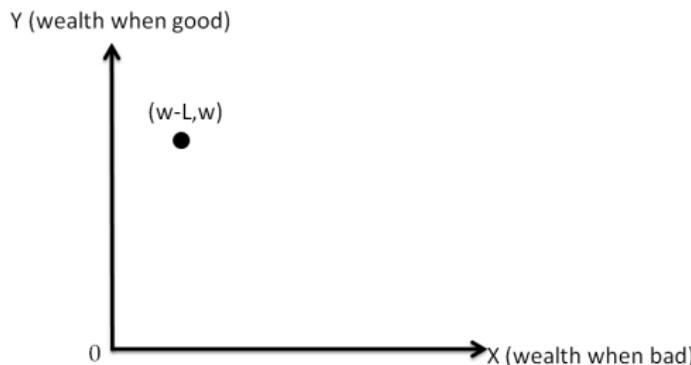
- ▶ We can formulate the problem of choosing an optimal consumption of insurance as the problem of choosing an optimal consumption bundle of two goods
- ▶ What are the two goods?
 1. Wealth when something bad happens X
 2. Wealth when nothing bad happens Y
- ▶ And the utility function $u(X, Y)$ is equal to the expected utility for the lottery $(X, p; Y, (1 - p))$ given the same Bernoulli utility function $v(\cdot)$

$$u(X, Y) = pv(X) + (1 - p)v(Y)$$

- ▶ Then what about the budget set?
- ▶ If you do not buy any insurance from Sam, what consumption bundle will you end up with?
 - ▶ $(w - L, w)$

Demand for Insurance

- ▶ First, without Sam and his offer of insurance, what is your budget set?
- ▶ Without the insurance, there is no way you can consume more of Y , wealth when bad things happen
- ▶ Therefore, without insurance, your budget set is a point $(w - L, w)$



Demand for Insurance

- ▶ How does Sam's insurance change the budget set?
- ▶ If you were to increase one unit of consumption of X (wealth in the bad state), how does your Y (wealth in the good state) change?
- ▶ To increase a unit of X , what do you need to do?
- ▶ How much insurance do you need to buy?
- ▶ If you buy z units of insurance of Sam, then
 - ▶ Your wealth in bad state: $w - L - qz + z$
 - ▶ Your wealth in good state: $w - qz$
- ▶ To increase one unit of X , we want $z - qz = 1$

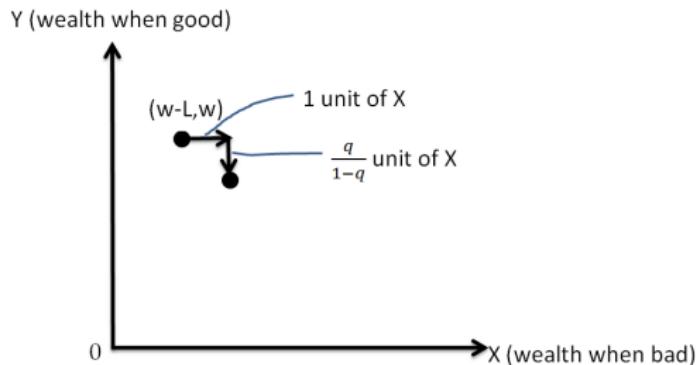
$$z = \frac{1}{1 - q}$$

Demand for Insurance

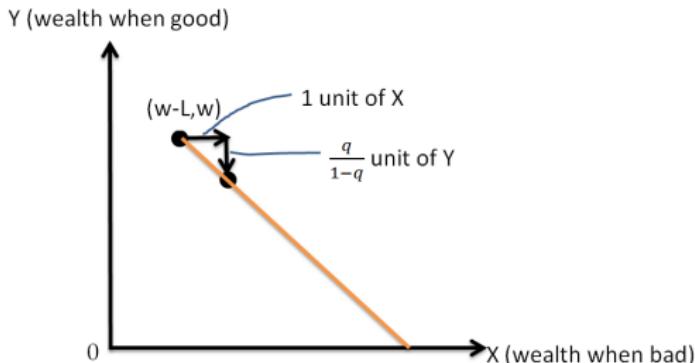
- ▶ So you need to buy $\frac{1}{1-q}$ units of insurance from Sam to increase one unit of X
- ▶ How much did your wealth in good state decrease because you bought $\frac{1}{1-q}$ units of insurance?
- ▶ So as a result of buying $\frac{1}{1-q}$ units of insurance, your consumption bundle changes as follows:
 - ▶ Your wealth in bad state: $w - L + 1$
 - ▶ Your wealth in good state: $w - \frac{q}{1-q}$

Drawing the Budget Set

- ▶ We have just found another point on the budget line
- ▶ We can connect these two dots to get the budget line

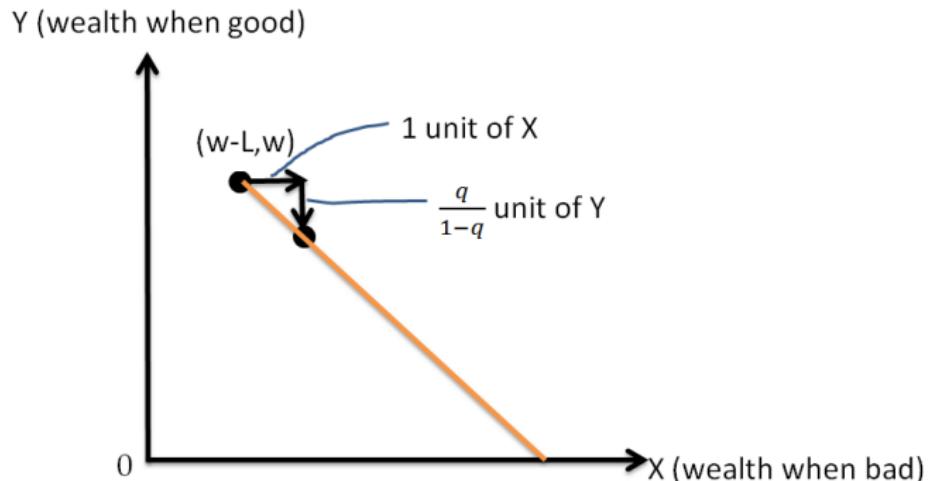


Drawing the Budget Set



- ▶ Why is there no line left of $(w-L, w)$?
 - ▶ Because Sam is not offering an insurance as follows:
 - ▶ "If you pay me r dollars now, I will pay you a dollar in the next period only if nothing bad happens"
 - ▶ There is no way for you to increase Y unless such insurance is offered

Drawing the Budget Set



Drawing the Budget Set

- ▶ What is the slope of the budget line?

$$\frac{\Delta Y}{\Delta X} = -\frac{q}{1-q}$$

- ▶ Remember the slope of the budget line in the consumer's problem: $-\frac{p_X}{p_Y}$
- ▶ So, in the insurance problem, the relative price between two goods is:

$$\frac{\text{Price of wealth in bad state}}{\text{Price of wealth in good state}} = \frac{q}{1-q}$$

Utility Function

- ▶ Now that we have the budget line, we only need to draw our indifference curve to find the optimal consumption bundle
- ▶ Our utility function $u(X, Y)$ is just the expected utility from the lottery $(X, p; Y, (1 - p))$ given the Bernoulli utility function $v(\cdot)$

$$u(X, Y) = pv(X) + (1 - p)v(Y)$$

Utility Function

- ▶ Suppose you are risk neutral. Then give me an example of $v(\cdot)$ that represents your preference

$$v(x) = x$$

- ▶ With this Bernoulli utility function, your utility function over the consumption bundle (X, Y) is

$$u(X, Y) = pX + (1 - p)Y$$

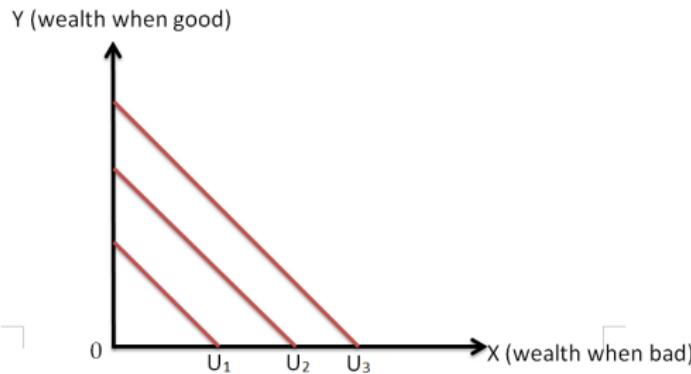
- ▶ With this utility function, how does your indifference curve look like?
- ▶ The indifference curve on which you get a utility level of u_1 is

$$pX + (1 - p)Y = u_1 \Rightarrow Y = -\frac{p}{1 - p}X + \frac{u_1}{1 - p}$$

- ▶ So the slope of the indifference curve is $\frac{p}{1-p}$

Indifference Curve for Risk Neutral DM

- ▶ Indifference curve for $u(X, Y) = pX + (1 - p)Y$



- ▶ Which indifference curve represents highest utility level?
- ▶ What is the marginal rate of substitution?

Optimal Consumption for Risk Neutral DM

- ▶ Now that we have
 - ▶ Budget line
 - ▶ Indifference curves
- ▶ We can find the optimal consumption bundle for risk neutral DM
- ▶ The optimal consumption bundle will depend on the slopes of the budget line and indifference curves
- ▶ The slope of the budget line $-\frac{q}{1-q}$ depends on q (decreases in q)
- ▶ The slope of the indifference curve $-\frac{p}{1-p}$ depends on p (decreases in p)

Optimal Consumption When $q > p$

- ▶ In this case, you do not buy any insurance
- ▶ In this case, a dollar in the bad state costs $\frac{q}{1-q}$ dollars in the good state
- ▶ However, since MRS is $\frac{p}{1-p}$, the value of a dollar in the bad state is worth $\frac{p}{1-p}$ times the one in the good state
- ▶ Since $\frac{q}{1-q} > \frac{p}{1-p}$, a dollar in the bad state is not worth its relative market price

Optimal Consumption When $q < p$

- ▶ In this case, you spend all your money on the insurance
- ▶ In this case, a dollar in the bad state costs $\frac{q}{1-q}$ dollars in the good state
- ▶ Since MRS is $\frac{p}{1-p}$, the value of a dollar in the bad state is worth $\frac{p}{1-p}$ times the one in the good state
- ▶ Since $\frac{q}{1-q} < \frac{p}{1-p}$, a dollar in the bad state is worth more than its relative market price

Optimal Consumption When $q = p$

- ▶ In this case, you are indifferent between buying any amount of insurance or not buying at all
- ▶ When $q = p$, then we say the price of insurance q is *actuarially fair*

Recap of the Lecture on March 22nd

- ▶ Typical situation where you might want insurance
 - ▶ Bernoulli utility function: $v(x)$
 - ▶ Initial wealth: w
 - ▶ With prob. p , suffers loss of L
 - ▶ With prob. $1 - p$, nothing happens
- ▶ Insurance offered
 - ▶ Pay q now, then you will get a dollar only when you suffer the loss
- ▶ Question: how much insurance will you want to purchase?

Recap of the Lecture on March 22nd

- ▶ We could have solved it algebraically
- ▶ Expected utility from buying z units of insurance is

$$pv(w - L - qz + z) + (1 - p)v(w - qz)$$

- ▶ Take the first order condition with respect to z
- ▶ Solve for z
- ▶ But instead we solved this problem by reformulating it with familiar graphical tools of economics: budget line and indifference curves

Recap of the Lecture on March 22nd

- ▶ What are the two "goods" to be consumed?
 - ▶ X : wealth in bad state
 - ▶ Y : wealth in good state
- ▶ What is your utility function over the two goods, $u(X, Y)$?

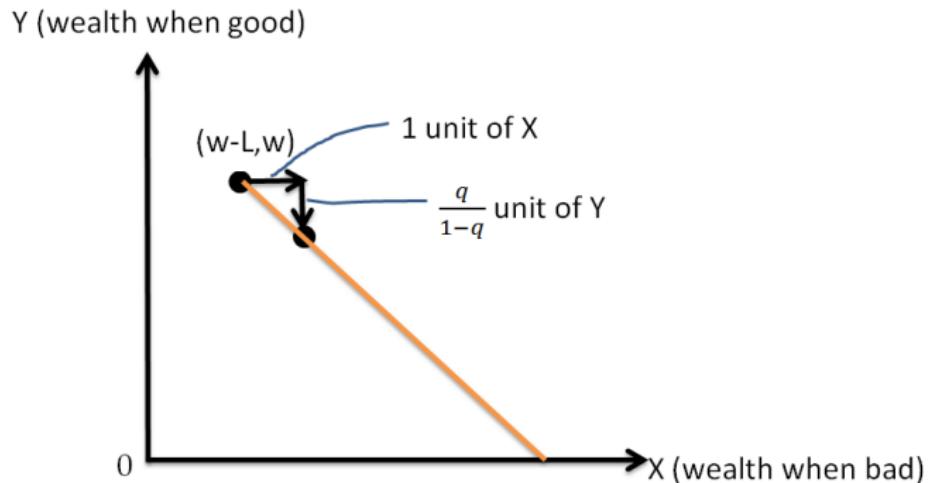
$$u(X, Y) = pv(X) + (1 - p)v(Y)$$

- ▶ How to draw budget line?
 - ▶ Calculate how much Y you should give up to get a unit (or one dollar) of X

Recap of the Lecture on March 22nd

- ▶ (X_0, Y_0) : your current consumption of the two goods
- ▶ If you want to increase your consumption of X from X_0 to $X_0 + 1$, what should you do?
 - ▶ You buy $\frac{1}{1-q}$ units of insurance
- ▶ Buying $\frac{1}{1-q}$ units of insurance decreases your consumption of Y by...
 - ▶ $\frac{q}{1-q}$ units (or dollars)
- ▶ So from (X_0, Y_0) to $(X_0 + 1, Y_0 - \frac{q}{1-q})$

Recap of the Lecture on March 22nd



Recap of the Lecture on March 22nd

- ▶ Risk neutral decision maker whose $v(x) = x$
- ▶ Utility function $u(X, Y) = pX + (1 - p)Y$
- ▶ Is his/her indifference curve
 1. Convex to the origin?
 2. Concave to the origin?
 3. Linear?
- ▶ The slope of each indifference curve is?
- ▶ Question: what does his/her optimal consumption bundle depend on?

Recap of the Lecture on March 22nd

- ▶ When $q > p$
- ▶ When $q < p$
- ▶ When $q = p$
 - ▶ When $q = p$, we say that the price of the insurance is...
 - ▶ *actuarially fair*
 - ▶ What is special about actuarially fair price? When $q = p$, what are insurer's expected profits from a unit of the insurance?
 - ▶ With probability p , the insurer has to pay out a dollar, so he suffers loss of $1-p$ dollars
 - ▶ With probability $(1-p)$, the insurer makes the profit of p dollars
 - ▶ So his expected profit is $-p(1-p)+(1-p)p=0$
 - ▶ So when the price of insurance is actuarially fair, the insurer makes expected profit of zero

Risk Neutral Averse Maker

- ▶ Now suppose you are a risk averse decision maker
- ▶ Most individuals should be risk averse
- ▶ Then what is the sign of the second derivative of your Bernoulli utility function?
- ▶ Your utility function over the consumption bundle (X, Y) is

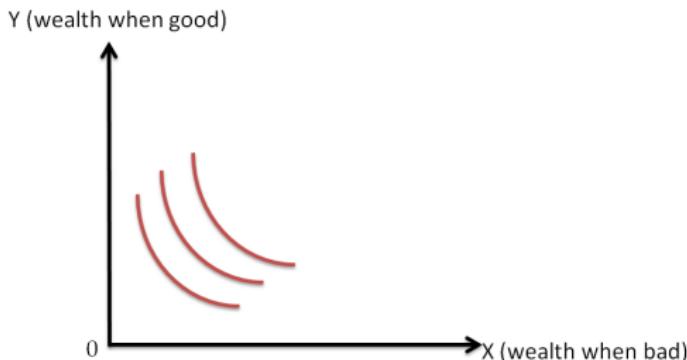
$$u(X, Y) = p v(X) + (1 - p) v(Y)$$

- ▶ MRS is equal to...

$$\frac{\frac{\partial u}{\partial X}}{\frac{\partial u}{\partial Y}} = \frac{p v'(X)}{(1 - p) v'(Y)}$$

- ▶ you can check for yourself that MRS decreases in X

Shape of the Indifference Curve for Risk Averse DM



- ▶ What is the optimal consumption bundle when the price of insurance is actuarially fair?
 - ▶ That is, $q = p$
- ▶ At the optimum, the $MRS = \text{slope of the budget line}$

$$\frac{pv'(X)}{(1-p)v'(Y)} = \frac{q}{1-q} = \frac{p}{1-p}$$

where the second equality comes from the price being actuarially fair

Shape of the Indifference Curve for Risk Averse DM

$$\frac{pv'(X)}{(1-p)v'(Y)} = \frac{p}{1-p}$$
$$\Rightarrow v'(X) = v'(Y) \Rightarrow X = Y$$

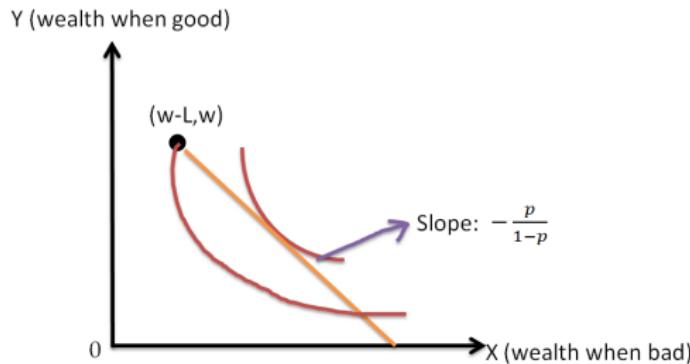
- ▶ This means that if the price of insurance is actuarially fair, risk averse DM optimally choose equalize X with Y
- ▶ How much insurance do you buy in the optimum? Suppose you buy z units and let's solve for z
- ▶ Then your wealth in the good state is $w - pz$
- ▶ Your wealth in the bad state is $w - pz - L + z$
- ▶ Then since in the optimum your wealth level across states is equalized,

$$w - pz = w - pz - L + z \Rightarrow z = L$$

- ▶ So you buy L units of insurance

Change in Expected Utility for Risk Averse DM

- ▶ We say that the risk averse DM is *fully insured* if $X=Y$
- ▶ Then is the risk averse DM better off by purchasing the insurance?



- ▶ Note that the expected wealth level decreases relative to status quo
 - ▶ Expected wealth without insurance: $w-pL$
 - ▶ Expected wealth with optimal level of insurance: $w-L$
- ▶ So by getting fully insured and removing risk (which was costly), risk averse DMs are better off

Optimal Consumption When $q > p$

- ▶ Again, at the optimum the MRS = the slope of the budget line

$$\frac{pv'(X)}{(1-p)v'(Y)} = \frac{q}{1-q}$$

- ▶ In this case, you do not get full insurance (i.e., $X=Y$) because if $X = Y$, this condition is not met
- ▶ When $X = Y$, the LHS (MRS) is smaller than RHS (the slope of the budget line)
- ▶ You can show graphically that at the optimum, $X < Y$
- ▶ This means that when $q > p$, risk averse DM is less than fully insured
- ▶ Intuition: bad event is not very likely, so the dollar in the bad state is not worth as much as when $q = p$, so you get less than full insurance

Market for Insurance

- ▶ So risk averse decision maker will demand for insurance because they get higher utility than by "removing" the risk with insurance
- ▶ Now let's look at the supply side of the insurance
- ▶ There can be two cases depending on bad events are independent across people or not
 1. Large number of insurees; the event of each suffering loss is independent
 2. Small number of insurees; the event of each suffering loss is NOT independent; large number of people immune to loss

Case 1: Large number of insurees; Independent Loss

- ▶ Let's recall what "independence of two events" mean
- ▶ Experiment: a procedure can be repeated infinitely, have a fixed set of outcomes
 1. Sam lives in his house
 2. Anujit lives in his house
- ▶ Sample space: the set of outcomes of an experiment
 1. {((Sam's house burns down), (Sam's house is intact))}
 2. {((Anujit's house burns down), (Anujit's house is intact))}
- ▶ An event: a subset of the sample space
- ▶ Two events A and B are independent if

$$Pr(A \cap B) = Pr(A)Pr(B)$$

Case 1: Large number of insurees; Independent Loss

- ▶ It is likely that the events (Sam's house burns down) and (Anujit's house burns down) are independent
 - ▶ Unless Sam and Anujit live next door or live together
 - ▶ We will talk about non-independent case
- ▶ Similarly, the event that an individual's house burns down is likely to be independent from the event that another individual's house burns down
- ▶ Suppose that the probability of each individual's house burning down is 0.005
- ▶ Each individual's house is worth \$1 mil.
- ▶ In case of fire, the value of house is zero

Case 1: Large number of insurees; Independent Loss

- ▶ Suppose that everyone's Bernoulli utility function is $v(x) = \sqrt{x}$
- ▶ Then, without insurance, the expected utility is

$$0.995 \times \sqrt{1,000,000} + 0.005 \times \sqrt{0} = 995$$

- ▶ Now suppose that ICBC offers a full insurance at the price of \$5,500
- ▶ What would be actuarially fair price?
- ▶ So ICBC is not offering an actuarially fair price
- ▶ As we have seen already, risk averse decision maker will get less than full insurance if the price is not actuarially fair
- ▶ However, for simplicity, let's assume that everyone purchases full insurance

Case 1: Large number of insurees; Independent Loss

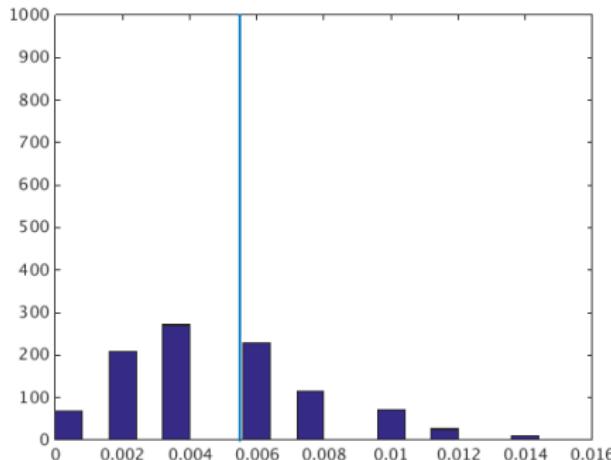
- ▶ Even without actuarially fair price, the insurees are better off with this full insurance because

$$\sqrt{1000000 - 5500} = 997.246$$

- ▶ Everyone's happy; but will ICBC make nonnegative profit so that it is willing to offer this insurance?
- ▶ Suppose there are 500 insurees. Then ICBC collects $\$5,500 \times 500 = \$2,750,000$
- ▶ If 3 or more house burns down (or the fraction of houses burnt down exceeds 0.55%), ICBC will make negative profits
- ▶ We can simulate the probability that at least three house burns down

Case 1: Large number of insurees; Independent Loss

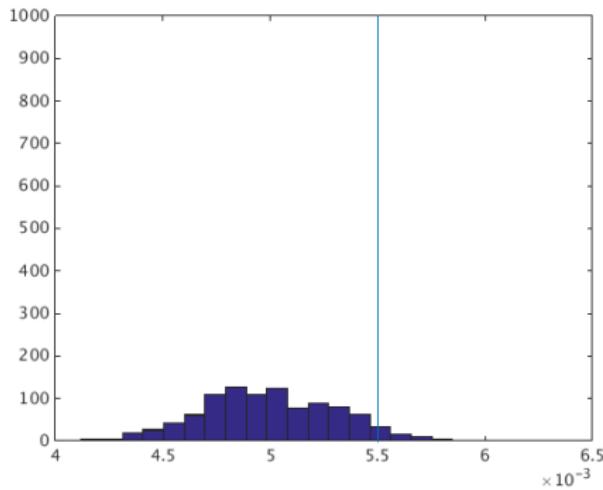
- With 500 insurees, the distribution of the fraction of population whose house burns down is



- $\Pr(\text{negative profit}) \approx 0.4540$
- What happens if we have more insurees?

Case 1: Large number of insurees; Independent Loss

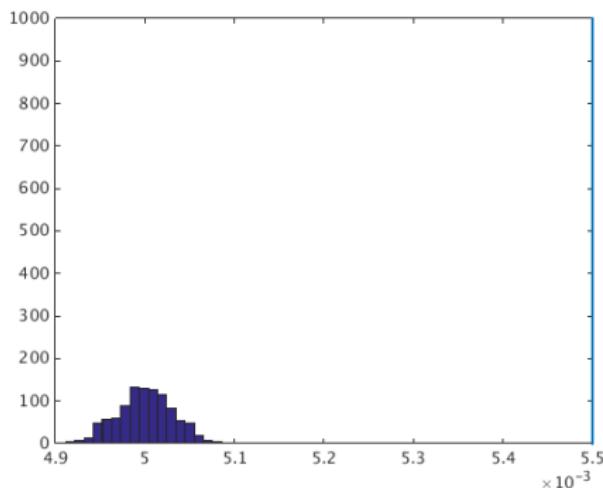
- With 50000 insurees, the distribution of the fraction of population whose house burns down is



- $\Pr(\text{negative profit}) \approx 0.051$
- What if more insurees?

Case 1: Large number of insurees; Independent Loss

- With 5000000 insurees (roughly the population of BC), the distribution of the fraction of population whose house burns down is



- $\Pr(\text{negative profit}) \approx 0$

Case 1: Large number of insurees; Independent Loss

- ▶ As we can see, as the number of insurees increases, the probability of negative profit gets closer to zero
- ▶ So as the pool of insurees becomes large, the insurer can be guaranteed to make positive profit
- ▶ Everyone who is insured is better off; the insurer makes positive profit
- ▶ Introducing insurance leads to *Pareto* improvement
 - ▶ Pareto improvement occurs when no one is worse off and someone is better off

Case 2: Small number of insurees; Non Independent Loss

- ▶ Some bad events hit a "region" or "neighborhood" in a larger society
 - ▶ Earthquake
 - ▶ Flood
 - ▶ Epidemic
- ▶ Suppose Sam and Anujit now lives in a city, say LA, where the probability of earthquake is 0.005
- ▶ The two events
 - ▶ Sam's house breaks down due to earthquake
 - ▶ Anujit's house breaks down due to earthquakeis not independent because, most likely

$Pr(\text{Sam \& Anujit's houses break down due to earthquake}) = 0.005$
rather than 0.005^2

Case 2: Small number of insurees; Non Independent Loss

- ▶ Suppose the value of all houses in LA is \$1,000,000
- ▶ Without insurance, the expected utility is 995 utils
- ▶ Now an insurer comes along and say,
- ▶ "Pay x dollars now; If earthquake, only then I will give you x dollars; otherwise I will keep the x dollars"
- ▶ Will anyone buy this insurance?
- ▶ The expected utility from this insurance is

$$\begin{aligned}& \sqrt{1000000 - x} \times 0.995 + \sqrt{1000000 - 1000000 - x} + x \times 0.005 \\&= 0.995\sqrt{1000000 - x}\end{aligned}$$

- ▶ which is less than 995 utils; so no one buys this insurance
- ▶ The reason this insurer cannot offer more than x dollars in case of Earthquake is because earthquake affects everyone in LA
- ▶ No demand for this kind of insurance, so no suppliers

Case 2: Small number of insurees; Non Independent Loss

- ▶ No private firm would be willing to provide insurance; but government intervention might help
- ▶ Government can appropriate money from non-LA citizens to cover the loss of LA citizens
- ▶ Suppose that there are 3 mil households in LA and the US consists of 300 mil households
- ▶ Assume that everyone outside of LA in the US has wealth of \$1 mil.
- ▶ The total loss from the earthquake to LA divided by the number of non-LA citizens is

$$\frac{3000000 \times \$1000000}{297000000} \approx \$10101$$

- ▶ So if everyone outside of LA pays \$10101 in case of earthquake, LA citizens are fully insured
- ▶ What happens to the expected utility of people in the LA and outside of LA?

Case 2: Small number of insurees; Non Independent Loss

- ▶ Without government intervention, the expected utility of
 - ▶ LA citizen: 995 utils
 - ▶ Non-LA citizens: 1000 utils
 - ▶ The sum of expected utility: $995 \times 3 \text{ mil} + 1000 \times 297 \text{ mil} = 299.985 \text{ tril utils}$
- ▶ With government intervention, the expected utility of
 - ▶ LA citizen: 1000 utils
 - ▶ Non-LA citizens: 999.97
 - ▶ The sum of expected utility: $1000 \times 3 \text{ mil} + 999.97 \times 297 \text{ mil} = 299.99109 \text{ tril utils}$
- ▶ So the sum of expected utility increased
- ▶ However, this is not a Pareto improvement (why?)

Case 2: Small number of insurees; Non Independent Loss

- ▶ Examples of government insurance
 - ▶ 9/11 Victim compensation fund
 - ▶ Medicaid (government insurance program in the US for low income families)
 - ▶ International donations for refugees
 - ▶ All kinds of disaster relief

Insurance Markets Are Welfare Improving

- ▶ For every risk in life, if full insurance is offered at the actuarially fair price, then everyone will benefit
- ▶ However, in the real world things are not that simple
 - ▶ There are risks for which not everyone is insured
 - ▶ Health insurance market in the US
 - ▶ Life insurance (the risk of death): as of 2015, 60 percent of Americans own life insurance
 - ▶ There are risks for which full insurance is not available
 - ▶ Deductibles: even after paying the premiums, we pay certain amount if we go see doctors
 - ▶ Caps on coverage: most insurance policies have caps on coverage
 - ▶ There are risks for which there exists no insurance
 - ▶ No insurance market for low income due to decline of the industry your job belongs to
 - ▶ No insurance market for marrying the wrong person

Why Insurance Markets Are Incomplete?

- ▶ There can be many reasons
 1. Some people cannot afford to buy insurance (e.g., health insurance)
 2. Some risks are uninsurable because everyone faces the same risk (e.g., global warming)
 3. Insurees know the likelihood of bad events while the insurer does not; leads to *adverse selection*
 4. Moral hazard: insurees will engage in "bad" behavior which makes insurers not want to offer insurance (e.g., eating junk food only after getting full health insurance)
- ▶ We are going to study how adverse selection leads to insurance market failure

Recap of the Lecture on March 24th

- ▶ Risk averse decision makers would choose to be fully insured if the price is fair
- ▶ Some losses are large (medical expenses for serious illnesses, accidents of expensive cars); why would anyone want to supply insurance
 - ▶ When large number of insurees with independent risk: as the pool gets large, the prob. of positive expected profit for the insurers converges to 1
 - ▶ Results in Pareto improvement
 - ▶ When small number of insurees with dependent risk: private insurers would not be able to provide insurance; government provides insurance
 - ▶ Not Pareto improvement; the sum of expected utility rises

Recap of the Lecture on March 24th

- ▶ Real world insurance markets are perfect
 - ▶ In some markets, there are people who are not insured at all (e.g., some not medically insured)
 - ▶ In other markets, full insurances are not provided (e.g., deductibles, cap on coverage)
 - ▶ For some risks there are no insurance that can be purchased in the market (e.g., risk of bad marriage)
- ▶ Reasons for imperfect insurance markets
 - ▶ Cannot afford insurance
 - ▶ Some risks are uninsurable
 - ▶ Moral hazard (bad behavior)
 - ▶ *Private information leading to adverse selection*

Model Without Private Information

- ▶ All insurees
 - ▶ have the same initial wealth w
 - ▶ have the same probability of loss p
 - ▶ If loss, then the same amount of loss L
 - ▶ have the same Bernoulli utility function $v(x)$; are risk averse

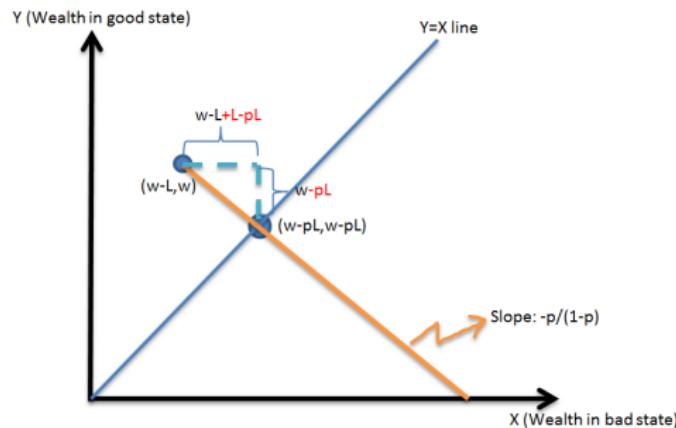
Model Without Private Information

- ▶ An insurance company can offer an insurance policy whose premium is $\$/I$ and whose benefit is $\$B$
 - ▶ The price of one dollar in the bad state would be $\frac{I}{B}$
- ▶ There is competition between insurance companies
- ▶ Question: what are the insurance policies that are offered in equilibrium?
- ▶ Definition of the equilibrium: in equilibrium
 1. No insurance policies that are offered makes negative profits
 2. No insurance policies that are not offered, if offered, would be more profitable than policies that are offered

Equilibrium in Model Without Private Info

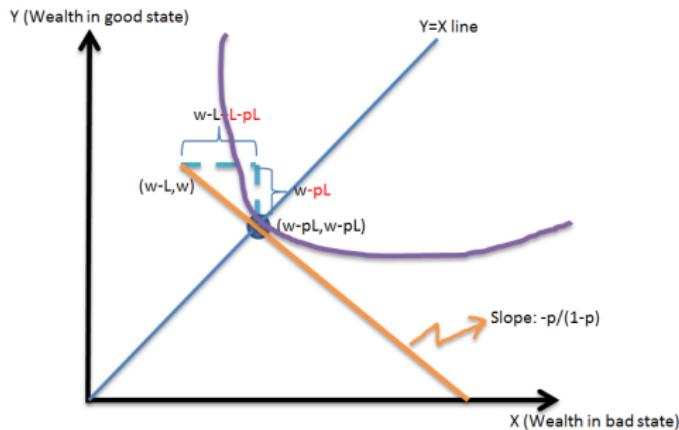
- ▶ We have seen that when the price is actuarially fair, then risk averse decision makers will get full insurance
- ▶ So our natural "guess" about the insurance policy (I^e, B^e) that is offered in equilibrium is the full insurance at the fair price
 - ▶ $B^e = L$ ("full insurance")
 - ▶ $I^e = pL$ ("at fair price")

Equilibrium in Model Without Private Info



- ▶ Indifference curve?

Equilibrium in Model Without Private Info



Equilibrium in Model Without Private Info.

- ▶ Let's verify if this is an equilibrium.
- ▶ Equilibrium Condition 1: Does (I^e, B^e) make nonnegative profits?
- ▶ What is the expected profit by insurance companies from offering (I^e, B^e) ?
 - ▶ Firms that offer this policy make the following expected profit:

$$p(I^e - B^e) + (1 - p)I^e = p(pL - L) + (1 - p)pL = 0$$

- ▶ So yes

Equilibrium in Model Without Private Info.

- ▶ Equilibrium Condition 2: are there any other policies that, if offered, make positive profits?
- ▶ Assume there is, in fact, a unoffered insurance policy (I, B) that would be more profitable than (I^e, B^e) if offered
- ▶ What is the expected profit from the policy (I, B) ?

$$p(I - B) + (1 - p)I = I - pB$$

- ▶ If (I, B) makes a positive profit, $I > pB$

Equilibrium in Model Without Private Info.

- Given an insurance policy (I, B) , the price of a dollar in the bad state is $\frac{I}{B}$ because

$$(\text{Premium}) = (\text{Benefit}) \times (\text{Price of a dollar of benefit})$$

$$\Rightarrow I = B \times \frac{I}{B}$$

- $I > pB$, this means that the price of one dollar in the bad state is more expensive than the fair price

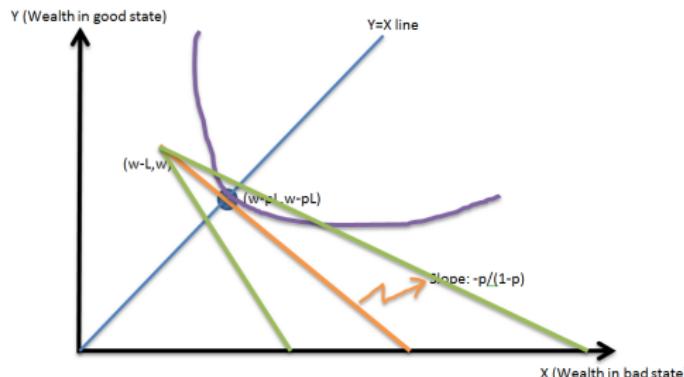
$$\frac{I}{B} > p$$

- Let q denote the price of a dollar in the bad state under the policy (I, B)

$$q = \frac{I}{B} > p$$

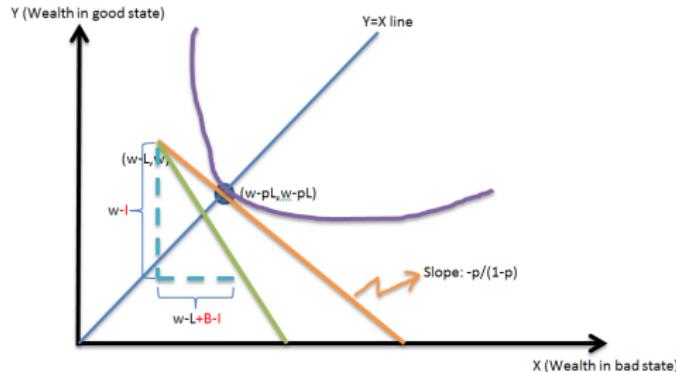
Equilibrium in Model Without Private Info.

- ▶ If one purchases (I, B) on which line his/her (X, Y) be?



- ▶ Because the price of a dollar in the bad state is higher than the fair price, the budget line would be...

Equilibrium in Model Without Private Info.



- ▶ What is the slope of the line that the policy (I, B) is on?

$$\frac{\Delta Y}{\Delta X} = \frac{-I}{B - I} = \frac{-qB}{B - qB} = -\frac{q}{1 - q}$$

- ▶ Will any consumers want to consume any policy on the line whose slope is $-\frac{q}{1-q}$? Why?

Equilibrium in Model Without Private Info.

- ▶ So $(I, B) = (qB, B)$ would be more profitable than $(I^e, B^e) = (pB, B)$ IF SOME CONSUMERS WERE ACTUALLY WILLING TO PURCHASE IT
- ▶ HOWEVER, NO ONE IS WILLING TO PURCHASE (I, B)
- ▶ Therefore, (I, B) will make no profit, hence not more profitable than the proposed equilibrium policy (I^e, B^e)
- ▶ So, our assumption that such (I, B) exists turns out to be wrong; there exists no policy that is more profitable than (I^e, B^e)
- ▶ Therefore, we have just verified that
 1. (I^e, B^e) makes nonnegative (zero) profit
 2. There is no unoffered policy that would make positive profit if offered
- ▶ Which means that we found an equilibrium where (I^e, B^e) is the only policy offered

Model with Private Information

- ▶ So far we have assumed that the probability that bad events occur is
 - ▶ p for everyone
 - ▶ It is known to the insurer
- ▶ From now on, we are going to relax these assumptions so that
 - ▶ there are two types of insurees: high-risk type and low risk type
 - ▶ High-risk type's probability of loss is p_h
 - ▶ Low-risk type's probability of loss is p_l
 - ▶ $p_h > p_l$
 - ▶ Insurers does not know the type of each insuree; it only knows that
 - ▶ In the population the fraction of high-risk type is λ and that of low-risk type is $1 - \lambda$

Model with Private Information

- ▶ Other components of the model is the same as before
 - ▶ Everyone has the same Bernoulli utility function $v(x)$
 - ▶ Everyone is risk averse
 - ▶ Initial wealth is $\$w$
 - ▶ The loss is equal to $\$L$
- ▶ There are many insurers, so there is competition between them
- ▶ An insurance policy consists of premium I and benefit B
- ▶ Question: what does the equilibrium of this insurance market look like?
- ▶ Again, in equilibrium,
 1. Whatever insurance policy that is offered should make nonnegative profits
 2. No policy outside of the offered policies exist that, if offered, would make a positive profit

Model with Private Information

- ▶ There may be two types of equilibria
 1. The one in which both types (high-risk and low-risk) buy the same insurance policy
 - ▶ We refer to this equilibrium as *pooling equilibrium*
 2. The one in which each risk type purchases different insurance policy
 - ▶ We refer to this equilibrium as *separating equilibrium*
- ▶ We do not know yet whether there exists a pooling/separating equilibrium, or both, or neither
- ▶ Let's see if there exists a pooling equilibrium
- ▶ There are three facts that are useful to know

Fact 1: at any (X, Y) , MRS of high type > MRS of low type

- ▶ This can be shown with algebra
- ▶ The utility function of high-risk type

$$p_h v(X) + (1 - p_h) v(Y)$$

- ▶ The utility function of low-risk type

$$p_h v(X) + (1 - p_h) v(Y)$$

Fact 1: at any (X, Y) , MRS of high type $>$ MRS of low type

- ▶ The MRS of high-risk type at some (X_0, Y_0)

$$\frac{p_h v'(X_0)}{(1 - p_h) v'(Y_0)}$$

- ▶ The MRS of low-risk type at the same (X_0, Y_0)

$$\frac{p_l v'(X_0)}{(1 - p_l) v'(Y_0)}$$

- ▶ Compare

$$\frac{p_h v'(X_0)}{(1 - p_h) v'(Y_0)} \text{ vs. } \frac{p_l v'(X_0)}{(1 - p_l) v'(Y_0)}$$

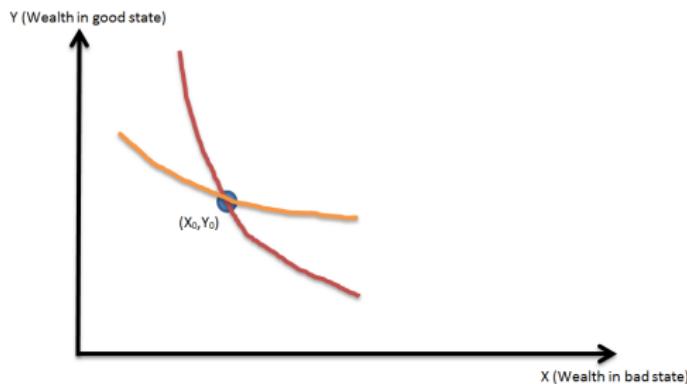
which one is larger?

Fact 1: at any (X, Y) , MRS of high type > MRS of low type

- ▶ But without resorting to algebra, we can reason why MRS is higher for high-risk type
- ▶ One interpretation of MRS: value of good X (in terms of good Y)
- ▶ Because high-risk type is more likely to end up in a bad state, the wealth in bad state is more valuable to him than to low-risk type
- ▶ Therefore, MRS of high risk type is larger

Fact 1: at any (X, Y) , MRS of high type $>$ MRS of low type

- ▶ If the MRS of high-type is larger than that of low-type at any given bundle (X_0, Y_0) , then the indifference curves of high type cut that of low type from above



Fact 2: The Actuarially Fair Price = $\lambda p_h + (1 - \lambda)p_l$

- ▶ In pooling equilibrium, only one policy (I^{pool}, B^{pool}) is offered, and all types buy this insurance policy
- ▶ What would be the price of a dollar in the bad state?

$$\frac{I^{pool}}{B^{pool}}$$

- ▶ What would be the price of a dollar in the bad state that makes the expected profit zero?
 - ▶ What would be the actuarially fair price?
- ▶ Remember when there was no asymmetric information and everyone's loss probability was the same, the fair price was...

$$p$$

which is equal to the probability of loss

- ▶ Alternatively, to insurers who offers a policy (I, B) , each insuree is a lottery

$$(I - B, p; I, (1 - p))$$

Fact 2: The Actuarially Fair Price = $\lambda p_h + (1 - \lambda) p_l$

- ▶ But with private information and heterogeneity in risk types, insurers do not know the probability of loss for each insuree
- ▶ Now, to insurers who offers a policy (I, B) , each insuree is a compound lottery (what was compound lottery?)
$$((I - B, p_h; I, (1 - p_h)), \lambda; (I - B, p_l; I, (1 - p_l)), 1 - \lambda)$$
- ▶ The expected profit from the compound lottery is
$$\lambda \times ((I - B)p_h + I(1 - p_h)) + (1 - \lambda) \times ((I - B)p_l + I(1 - p_l))$$
- ▶ You can check that to make the expected profit zero, it must be that

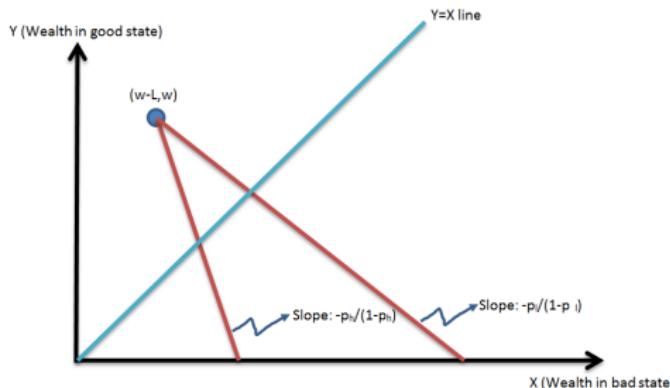
$$\bar{p}B = I$$

where \bar{p} is the average of the loss probabilities weighted by the fraction of risk types

$$\bar{p} = \lambda p_h + (1 - \lambda) p_l$$

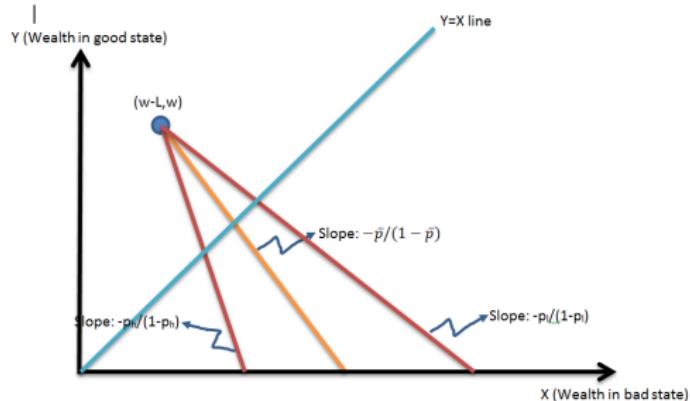
Fact 2: The Actuarially Fair Price = $\lambda p_h + (1 - \lambda)p_l$

- Policies (I, B) such that $\frac{I}{B} = p_h$ and $\frac{I}{B} = p_l$ correspond to points on these lines



- Where would the line whose slope is $-\frac{\bar{p}}{1-\bar{p}}$ be drawn?

Fact 2: The Actuarially Fair Price = $\lambda p_h + (1 - \lambda)p_l$



- Any insurance policy with actuarially fair price correspond to a point on the yellow line

Fact 3: The Equilibrium Insurance Policy Makes Zero Profit

- ▶ Let (I^{pool}, B^{pool}) denote the only policy that is offered in the pooling equilibrium (if it exists)
- ▶ It must be that any firm that offers this policy should make zero profit
- ▶ Let's assume that (I^{pool}, B^{pool}) in fact makes positive expected profits, that is,

$$I^{pool} > \bar{p}B^{pool}$$

- ▶ This means that the price of a dollar in the bad state is higher than fair, i.e., $\frac{I^{pool}}{B^{pool}} > \bar{p}$
- ▶ Then there is another policy (I, B) that would be more profitable than (I^{pool}, B^{pool})

Fact 3: The Equilibrium Insurance Policy Makes Zero Profit

- ▶ The idea: lower the premium (but not the benefit) just a tiny bit so that
 1. The profit margin is almost the same
 2. Everyone will prefer (I, B) to (I^{pool}, B^{pool}) because of lower premium, hence captures the entire market
- ▶ Similar to how Bertrand Competition pushes down prices to the marginal cost
- ▶ But this would violate the equilibrium condition 2, which is...
 - ▶ There is no unoffered policy that, if offered, would be more profitable than the equilibrium policy
- ▶ This contradicts our assumption that (I^{pool}, B^{pool}) is an equilibrium policy
- ▶ Therefore, our assumption that led to this contradiction (that (I^{pool}, B^{pool}) makes a positive profit) must be wrong

Fact 3: The Equilibrium Insurance Policy Makes Zero Profit

- ▶ The fact that (I^{pool}, B^{pool}) should make zero expected profit means that the price of a dollar in the bad state

$$\frac{I^{pool}}{B^{pool}}$$

is equal to...

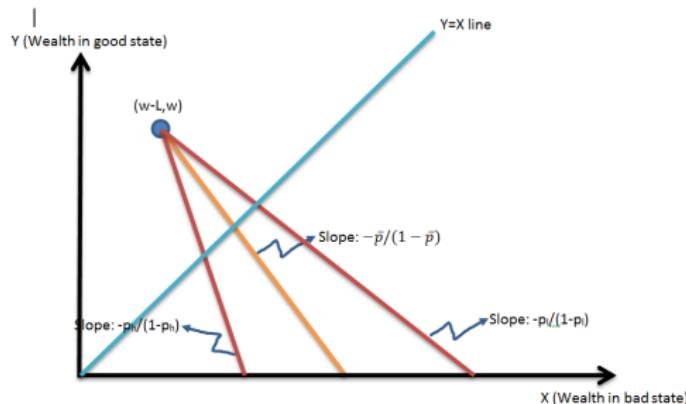
- ▶ the actuarially fair price, $\bar{p} = \lambda p_h + (1 - \lambda)p_l$

Pooling Equilibrium

- ▶ To sum up, three facts we can use to see whether there is a pooling equilibrium or not
 1. MRS of high risk type > MRS of low risk type
 2. Actuarially fair price $\bar{p} = \lambda p_h + (1 - \lambda)p_l$
 3. The equilibrium policy (I^{pool}, B^{pool}) makes zero profits

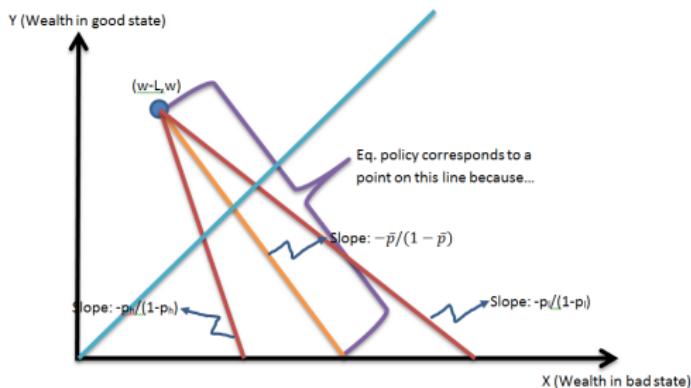
Pooling Equilibrium

- Actuarially fair price $\bar{p} = \lambda p_h + (1 - \lambda)p_l$



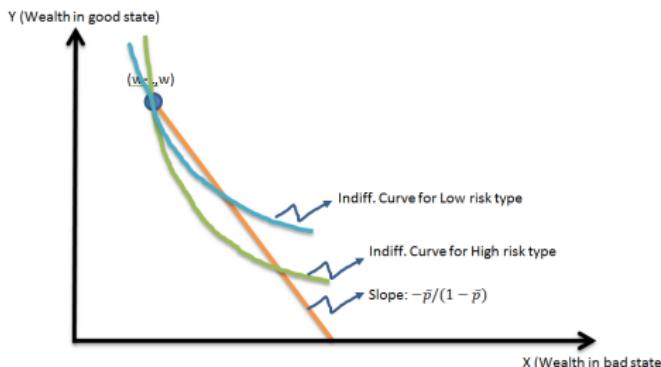
Pooling Equilibrium

- ▶ Actuarially fair price $\bar{p} = \lambda p_h + (1 - \lambda)p_l$
- ▶ The equilibrium policy (I^{pool}, B^{pool}) makes zero profits



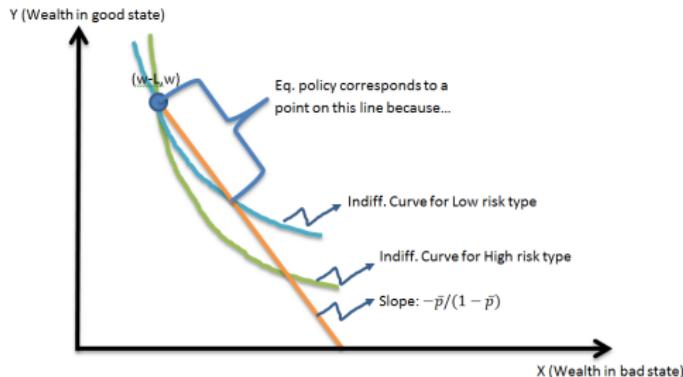
Pooling Equilibrium

- ▶ Let's draw indifference curve for each type that passes through the no-insurance point $(w - L, w)$
- ▶ We use the fact that MRS of high risk type $>$ MRS of low risk type



Pooling Equilibrium

- ▶ This further narrow the set of policies that can be offered in the pooling equilibrium because...



- ▶ Both types should be at least as well-off with insurance as when they have no insurance

Recap of the Lecture on March 29th

- ▶ Again, X : wealth in the bad state
- ▶ Y : wealth in the good state
- ▶ (X, Y) will be referred to as a consumption bundle

Recap of the Lecture on March 29th

- ▶ If you do not buy any insurance, $X = w - L$ and $Y = w$
- ▶ Let's refer to this point as "no insurance point" or "endowment point"

Recap of the Lecture on March 29th

- ▶ Your utility function is

$$pv(X) + (1 - p)v(Y)$$

- ▶ You are risk averse

Recap of the Lecture on March 29th

- ▶ An insurance policy consists of (I, B)
- ▶ Each consumption bundle can be achieved with only one insurance policy
- ▶ So when we refer to a point on the X-Y plane, we are going to use the term "consumption bundle" and "insurance policy that achieves that consumption bundle" interchangeably

Recap of the Lecture on March 29th

- ▶ If endowment is $(w - L, w)$ and you purchase an insurance policy (I, B) , then what would be your consumption bundle after the purchase?
 - ▶ X?
 - ▶ Y?
- ▶ Your Y decreases by I ; your X increases by $B-I$
- ▶ Under insurance policy (I, B) , how much of Y should you give up to increase a dollar of X?
- ▶ $\frac{I}{I-B}$: will be referred to as "exchange rate"

Recap of the Lecture on March 29th

- ▶ Any insurance policy on the line that connects the endowment point and a given insurance policy (I, B) has the same exchange rate as (I, B)
- ▶ $-\frac{I}{B-I}$ is also the slope of this line
- ▶ exchange of an insurance policy $(I, B) = -$ slope of the line that connects it with the endowment point

Recap of the Lecture on March 29th

- ▶ When there is no asymmetric information, p is known to everyone
- ▶ We showed that the only insurance policy that will be offered in equilibrium is the full insurance (I^e, B^e) such that

$$I^e = pB^e$$

- ▶ What are the conditions for equilibrium?
 1. Offered policy should make nonnegative expected profit
 2. Unoffered policy, if offered (IN ADDITION TO THE OFFERED POLICIES), should not make more profit than the offered ones

Recap of the Lecture on March 29th

- ▶ Does (I^e, B^e) such that $I^e = pB^e$ satisfy the equilibrium conditions?
- ▶ The first condition is satisfied because the expected profit from (I^e, B^e) is

$$I^e - pB^e = pB^e - pB^e = 0$$

- ▶ Now let's verify the second condition
- ▶ An alternative policy (I, B) provides the expected payoff of $I - pB$
- ▶ Insurance companies will want to offer this alternative if it is more profitable than (I^e, B^e) . That is,

$$I - pB > I^e - pB^e = 0$$

- ▶ But for this alternative to make profit, it has to be demanded by some people when both it and (I^e, B^e) are offered simultaneously; will it?

Recap of the Lecture on March 29th

- ▶ The exchange rate of (I^e, B^e)

$$\frac{I^e}{B^e - I^e} = \frac{p}{1 - p}$$

- ▶ The exchange rate of (I, B)

$$\frac{I}{B - I} > \frac{p}{1 - p} \text{ (Why?)}$$

- ▶ (I, B) is a bad deal (has to give up too much Y to get a dollar of X)
- ▶ No consumers would want to switch to (I, B) ; makes no sales/profits (show with plots)
- ▶ So there is no alternative policies that are more profitable
- ▶ (I^e, B^e) is the equilibrium policy when there is no asymmetric information

Recap of the Lecture on March 29th

- ▶ Then we introduced private information
- ▶ There are two types:
 1. High risk type p_h
 2. Low risk type p_l
 3. $p_h > p_l$
- ▶ Insurance companies cannot tell which type individual consumer is
- ▶ They only know that the fraction of population who is high risk type is λ

Recap of the Lecture on March 29th

- ▶ There may be two kinds of equilibria
 1. Pooling: only one policy is offered; all types purchase it
 2. Separating: two policies are offered; each risk type purchases different policies
- ▶ Let's see if there can be a pooling equilibrium
- ▶ That is, let's see if there can be one policy such that
 1. Insurance firms that offer it make nonnegative profits
 2. No unoffered policies exist that, if offered, would be more profitable than the offered one

Recap of the Lecture on March 29th

- ▶ Three useful facts when checking if there exists a pooling eq.
 1. MRS of high risk type > MRS of low risk type
 2. The actuarially fair price = the average probability of loss
 3. The equilibrium policy (I^{pool}, B^{pool}) should make zero profit

Recap of the Lecture on March 29th

- ▶ Fact 1: MRS of high risk type $>$ MRS of low risk type
 - ▶ MRS is the value of good X in terms of good Y
 - ▶ Why would high risk type value X more highly than low risk type?
 - ▶ Because they are more likely to suffer loss

Recap of the Lecture on March 29th

- ▶ Fact 2: The actuarially fair price = the average probability of loss
 - ▶ Under policy (I, B) , the price of a dollar of benefit = $\frac{I}{B}$
 - ▶ Not to be confused with the exchange rate (how much X one needs to give up to get a dollar of Y) = $\frac{I}{B-I}$
- ▶ Actuarially fair price = the price of a dollar of benefit that makes the expected profit zero

Recap of the Lecture on March 29th

- ▶ Suppose an insurance company sells a policy (I, B) to a random person on the street without knowing his/her risk type
- ▶ What is the expected profit?
- ▶ Prob. that this random person is high risk: λ
- ▶ If high type, then the policy for that person is like a lottery

$$(I - B, p_h; I, (1 - p_h))$$

- ▶ Prob. that this random person is low risk: $(1 - \lambda)$
- ▶ If low type, then the policy for that person is like a lottery

$$(I - B, p_l; I, (1 - p_h))$$

Recap of the Lecture on March 29th

- ▶ Selling the policy to a random person without knowing her types is like getting a compound lottery

$$((I - B, p_h; I, (1 - p_h)), \lambda; (I - B, p_l; I, (1 - p_h)), 1 - \lambda)$$

- ▶ The expected profit from this compound lottery is

$$\begin{aligned} & \lambda \times \text{expected profit from } (I - B, p_h; I, (1 - p_h)) \\ & + (1 - \lambda) \times \text{expected profit from } (I - B, p_l; I, (1 - p_h)) \end{aligned}$$

- ▶ You can verify that the price of a dollar of benefit that makes this expected profit zero equals \bar{p}

$$\bar{p} = \lambda p_h + (1 - \lambda) p_l$$

which is the average probability of loss

Recap of the Lecture on March 29th

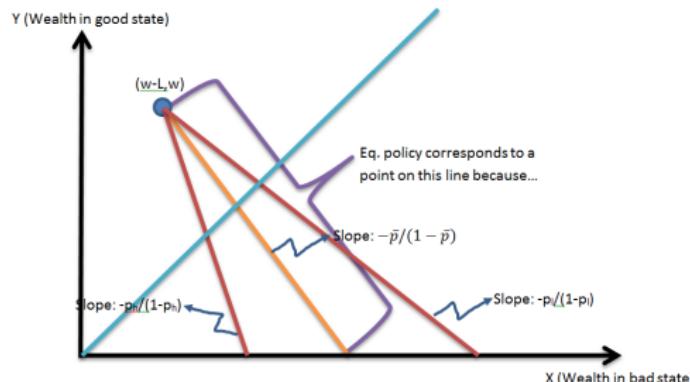
- ▶ Fact 3: Policy (I^{pool}, B^{pool}) offered in pooling equilibrium should make zero profit
- ▶ Why is this true?
- ▶ Let's assume it is not true
- ▶ Then there would be an alternative policy that, if offered, would be more profitable than (I^{pool}, B^{pool})
 - ▶ Slightly lower the premium to capture the entire demand
- ▶ Then the second equilibrium condition would be violated
- ▶ But wait, isn't (I^{pool}, B^{pool}) suppose to satisfy both equilibrium conditions? How can the second condition be violated?
- ▶ The only reason that the second condition is violated is because our assumption that I^{pool}, B^{pool} makes positive profit is wrong

Recap of the Lecture on March 29th

- ▶ Now we are ready to check whether there is a pooling equilibrium
- ▶ First, let's use Fact 3: equilibrium policy makes zero expected profits
- ▶ For any policies to make zero profits, what should the price of a dollar of benefit be?
- ▶ Now we use Fact 2: the actuarially fair price = the average probability of loss
- ▶ So any policies under which the price of a dollar of benefit is \bar{p} can potentially be equilibrium policies

Recap of the Lecture on March 29th

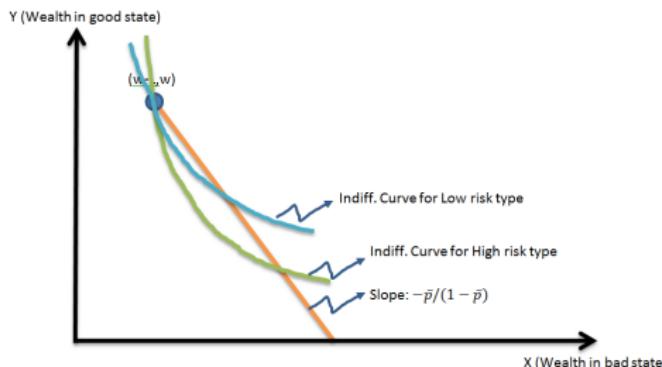
- ▶ Let's plot insurance policies under which the price of a dollar of benefit is \bar{p}
- ▶ If the price of a dollar of benefit is \bar{p} , then the exchange rate is $\frac{\bar{p}}{1-\bar{p}}$
- ▶ Because exchange rate $= \frac{I}{B-I} = \frac{\bar{p}B}{B-\bar{p}B} = \frac{\bar{p}}{1-\bar{p}}$
- ▶ The line whose slope is - exchange rate and passes through the endowment point



- ▶ Any insurance policies on this line have the price of a dollar of benefit at \bar{p}

Recap of the Lecture on March 29th

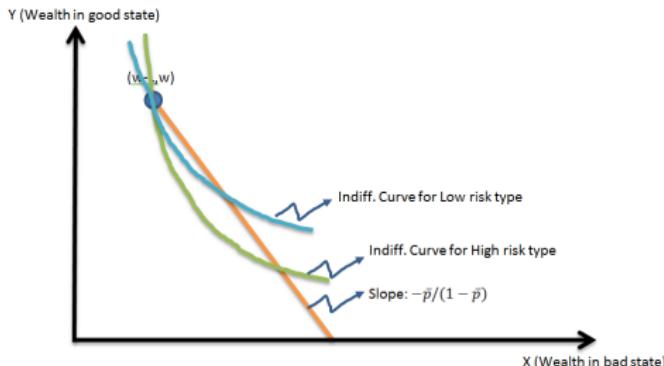
- ▶ Let's draw indifference curve for each type that passes through the no-insurance point $(w - L, w)$
- ▶ We can use the Fact 1: MRS of high risk type $>$ MRS of low risk type



Recap of the Lecture on March 29th

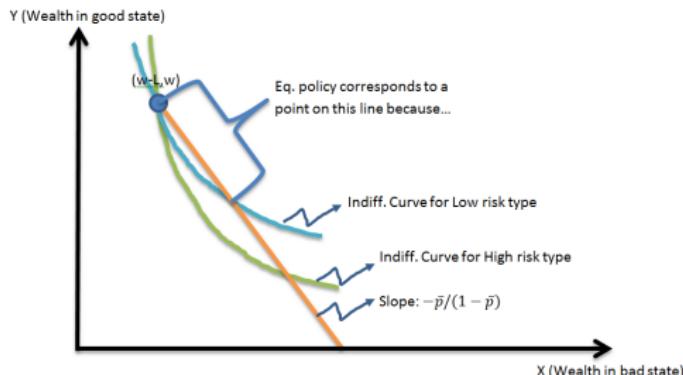
- ▶ Everyone, regardless of their types, should be at least as well-off when they purchase (I^{pool}, B^{pool}) as when they have no insurance
- ▶ Why? If they are worse off with (I^{pool}, B^{pool}) than no insurance, no one will buy the policy
- ▶ This means that (I^{pool}, B^{pool}) should be
 1. on the orange line (i.e., makes zero expected profit)
 2. gives both types the same or higher expected utility than no insurance point

Recap of the Lecture on March 29th



- ▶ Where on the orange line are the insurance policies that give both types the same or higher expected utility than no insurance point?

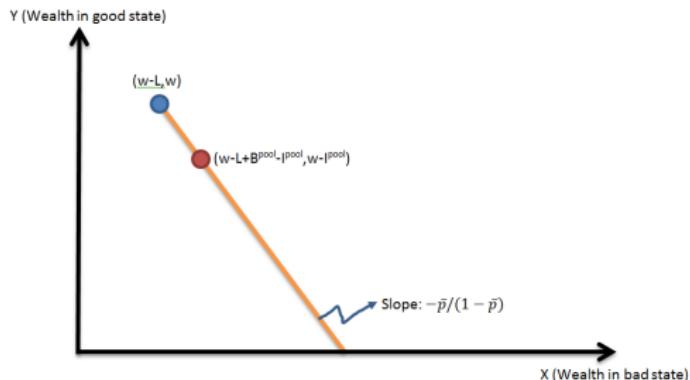
Recap of the Lecture on March 29t



- Let's choose any policy on this segment of the orange line and see if it can be a pooling equilibrium

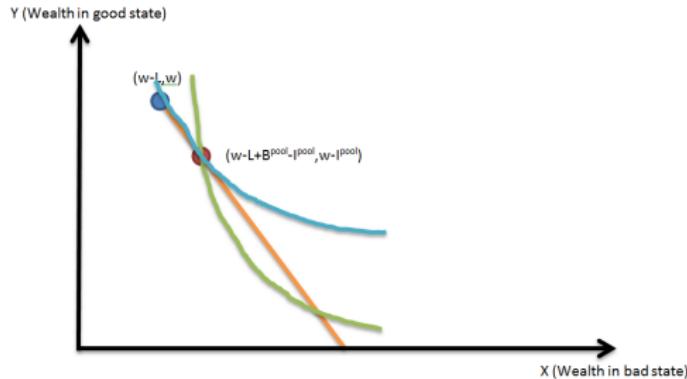
Pooling Equilibrium

- ▶ A candidate for the policy offered in a pooling equilibrium



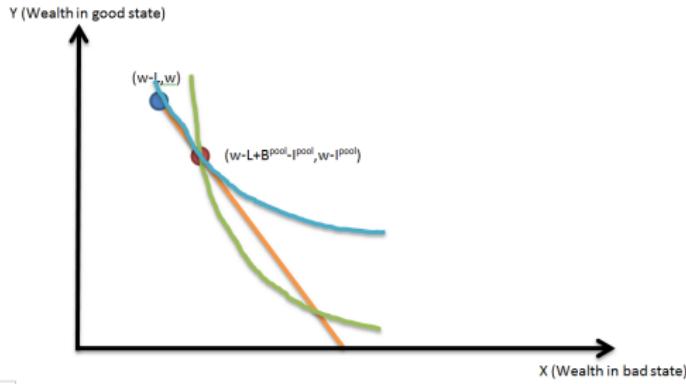
Pooling Equilibrium

- ▶ The indifference curves that pass through the insurance policy



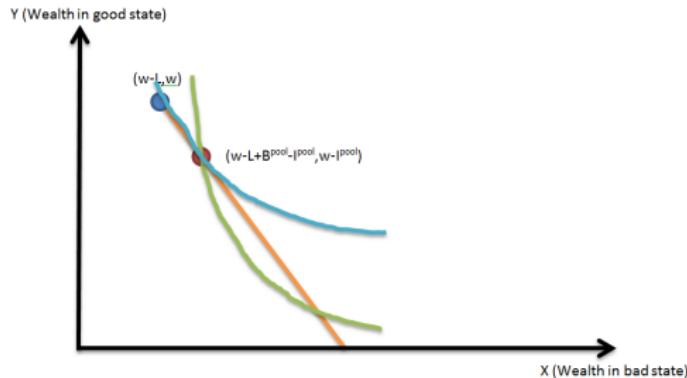
- ▶ Which one is the indiff. curve for high risk type? Why?

Pooling Equilibrium



- ▶ Is this an equilibrium?
- ▶ Remember the equilibrium conditions
 1. All policies offered in equilibrium make nonnegative profits
 2. No unoffered policies, if offered, make positive profits
- ▶ Does (I^{pool}, B^{pool}) make nonnegative profits?
- ▶ Yes, because...

Pooling Equilibrium

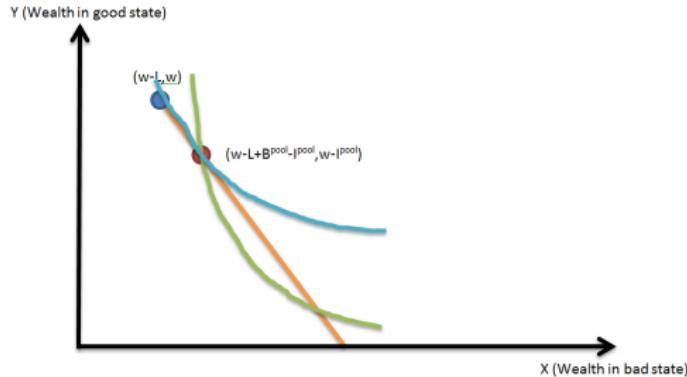


- Now we need to check whether there is any unoffered policy that makes positive profits when offered
- If there is, then (I^{pool}, B^{pool}) is not an equilibrium insurance policy

Pooling Equilibrium

- ▶ Spoiler: yes, there is an alternative policy that is more profitable than (I^{pool}, B^{pool}) if offered
- ▶ But more important, how to find such an alternative policy?
- ▶ We could try all insurance policies
- ▶ Instead, we can examine our candidate policy (I^{pool}, B^{pool}) closely and think about how to come up with more profitable alternatives

Pooling Equilibrium



- ▶ We know that the total profit from selling (I^{pool}, B^{pool}) is zero
- ▶ We can decompose the total profits into two parts
 1. Profits from high risk types
 2. Profits from low risk types

even though types are not observable

Pooling Equilibrium

- ▶ Are the firms making positive/negative profits from high risk types?
- ▶ The expected profits from selling (I^{pool}, B^{pool}) to a high risk type (even though the types are not observable, we can still calculate this)
 - ▶ With prob. p_h , the high risk type will suffer loss, so the firm gets $I^{pool} - B^{pool}$
 - ▶ With prob. $1 - p_h$, no loss, so the firm gets I^{pool}
- ▶ So the expected profit is

$$p_h(I^{pool} - B^{pool}) + (1 - p_h)I^{pool} = I^{pool} - p_hB^{pool}$$

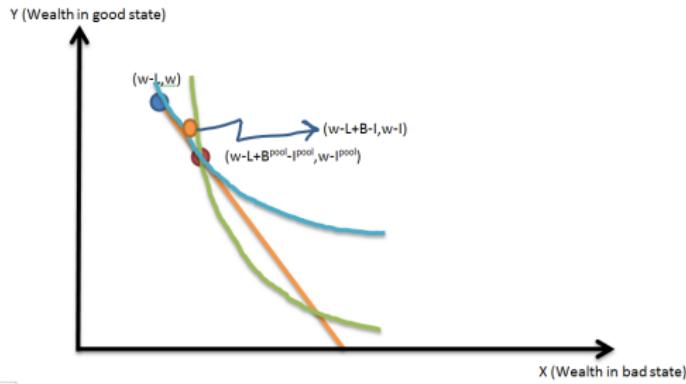
- ▶ The expected profit is negative (why?)
- ▶ So the firms are making negative profit on high risk types

Pooling Equilibrium

- ▶ Similarly, are the firms making positive/negative profits from low risk types?
- ▶ You can check for yourself that the firms are making positive profits from low risk types
- ▶ Firms are willing to offer (I^{pool}, B^{pool}) / making zero profit because *gains from low risk types offset loss from high risk types*
- ▶ This gives you an idea of how to find more profitable policies
- ▶ If we can offer an alternative policy (I, B) such that
 1. can attract ONLY low risk type when simultaneously offered with (I^{pool}, B^{pool})
 2. is not very different from (I^{pool}, B^{pool}) , so the alternative makes positive profits on low risk types

then such policy would likely be more profitable than (I^{pool}, B^{pool})

Pooling Equilibrium



- ▶ Consider an insurance policy (I, B) that corresponds to the orange dot
- ▶ When both orange and red are offered, which policy does high risk type prefer?
- ▶ When both orange and red are offered, Which policy does low risk type prefer?

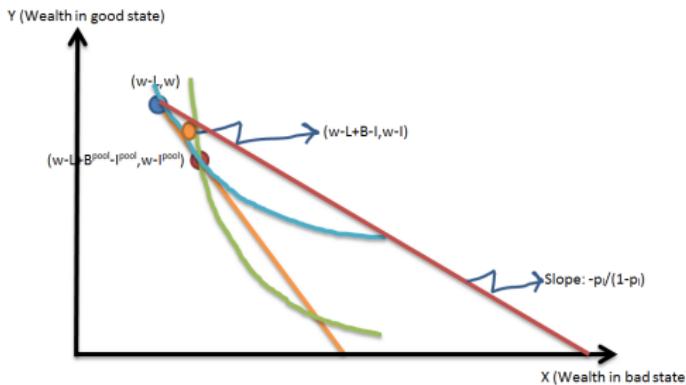
Pooling Equilibrium

- ▶ Only the low risk type will prefer (I, B) to (I^{pool}, B^{pool})
- ▶ If this policy makes positive profit, then (I^{pool}, B^{pool}) is not an equilibrium
- ▶ What is the expected profits from (I, B) ?
- ▶ Is it $\bar{p}(I - B) + (1 - \bar{p})I$?
- ▶ NO, because only low risk type is attracted to (I, B) , so the insurer who offers (I, B) knows that the prob. of loss is p_I for whoever buys the policy
- ▶ Therefore, the expected profit is

$$p_I(I - B) + (1 - p_I)I = I - p_I B$$

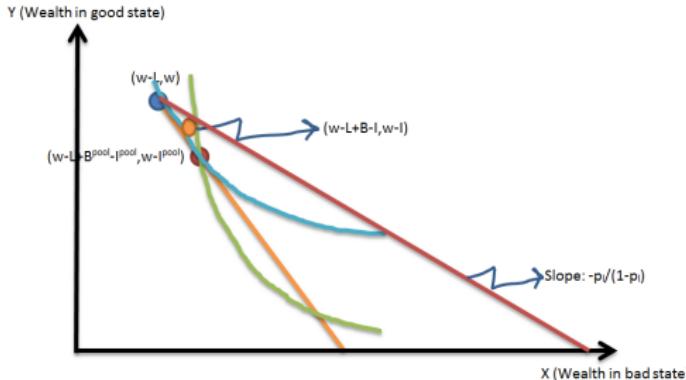
- ▶ As long as (I, B) is such that $I - p_I B > 0$, then (I, B) earns positive profits

Pooling Equilibrium



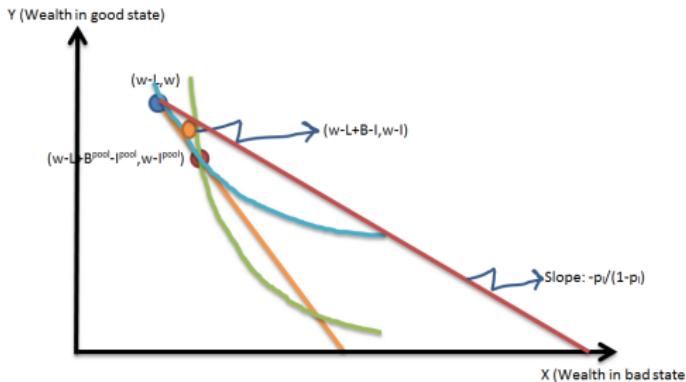
- ▶ Suppose that the orange dot is below the red line
- ▶ The slope of the red line is $-\frac{p_l}{1-p_l}$
- ▶ What would be the expected profit of policies on the red line, when sold only to low risk types?
 - ▶ It is zero; can be shown algebraically (homework)
- ▶ The red line is the zero-profit line for low risk types

Pooling Equilibrium



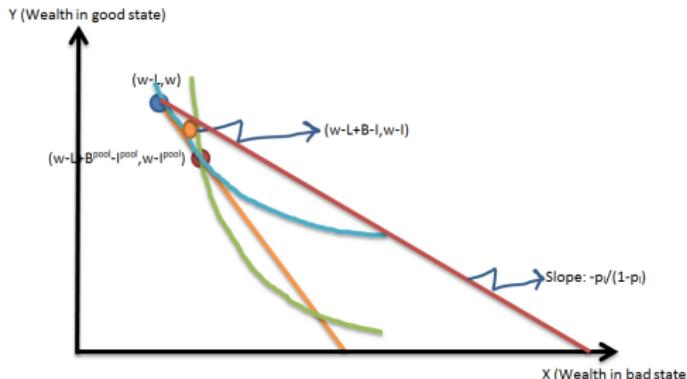
- ▶ Would policies that are BELOW the red line, such as orange dot, make positive profits if sold only to low risk types?
 - ▶ The answer is yes; can be shown algebraically (homework)
- ▶ Because the red line is the zero-profit line for low risk types, policies that are below the line earn positive profits

Pooling Equilibrium



- ▶ We have just shown that there exists an alternative policy that, if offered simultaneously with (l^{pool}, B^{pool}) , will be more profitable
- ▶ Therefore, (l^{pool}, B^{pool}) is/is not an equilibrium?

There is No Pooling Equilibrium



- ▶ Intuitively, why does the pooling equilibrium fail?
- ▶ It's because low risk types are paying too much per dollar of benefit: if types are fully observable, the fair price for low risk types is p_l
- ▶ Under the candidate pooling policy, low risk types is paying $\bar{p} > p_l$
- ▶ So low risk types want to pay less per dollar of benefit, even if it means less coverage
- ▶ Insurance firms "cream-skim" these low risk types

Health Insurance in British Columbia

- ▶ Pooling equilibrium where everyone purchases the same policy would not exist if insurances are supplied by profit maximizing firms
- ▶ But it is possible if it is mandated by law as in BC
- ▶ Medical Services Plan (MSP)

Home / Health / Health & Drug Coverage / Medical Services Plan (MSP) /

B.C. Residents

Under the *Medicare Protection Act*, enrolment with the Medical Services Plan (MSP) is mandatory for all eligible residents and their dependents.

Eligibility and Enrolment

Learn the definition of a resident, about coverage for dependents, and apply for enrolment in MSP.

» [Eligibility and Enrolment](#)

Benefits

MSP covers the cost of medically required services provided by physicians and supplementary health care practitioners. Learn more about benefits provided by MSP as well as the services that are excluded.

» [Benefits](#)

Premiums

Premiums are required by MSP. Learn about payment options available to you, including premium assistance

» [Premiums](#)

- » [B.C. Residents](#)
- » [Eligibility and Enrolment](#)
- » [Benefits](#)
- » [Premiums](#)

Health Insurance in British Columbia

- ▶ If MSP were to be financially sustainable, it has to make nonnegative profits
- ▶ Unlikely to provide full insurance; otherwise unlikely to break even

Medical Benefits

The Medical Services Plan (MSP) provides the following benefits:

- medically required services provided by a physician enroled with MSP;
- maternity care provided by a physician or a midwife;
- medically required eye examinations provided by an ophthalmologist or optometrist;
- diagnostic services, including x-rays, provided at approved diagnostic facilities, when ordered by a registered physician, midwife, podiatrist, dental surgeon or oral surgeon;
- dental and oral surgery, when medically required to be performed in hospital (excluding restorative services, i.e.: fillings, caps, crowns, root canals, etc.)*;
- orthodontic services related to severe congenital facial abnormalities.

- ▶ MSP will cover medically required services you receive from physicians and midwives

Health Insurance in British Columbia

Services Not Covered by MSP

MSP does not provide coverage for the following:

- services that are deemed to be not medically required, such as cosmetic surgery;
- dental services, except as outlined under benefits;
- routine eye examinations for persons 19 to 64 years of age;
- eyeglasses, hearing aids, and other equipment or appliances;
- prescription drugs (see PharmaCare);
- acupuncture, chiropractic, massage therapy, naturopathy, physical therapy and non-surgical podiatry services (except for MSP beneficiaries with premium assistance status);
- preventive services and screening tests not supported by evidence of medical effectiveness (for example, routine annual "complete" physical examinations, whole body CT scans, prostate specific antigen (PSA) tests);
- services of counsellors or psychologists;
- medical examinations, certificates or tests required for:
 - driving a motor vehicle
 - employment
 - life insurance
 - school or university
 - recreational and sporting activities
 - immigration purposes

- ▶ Services that are not medically required are not covered

Health Insurance in British Columbia

Premiums

In B.C., premiums are payable for MSP coverage and are based on family size and income.

Monthly Premium Rates

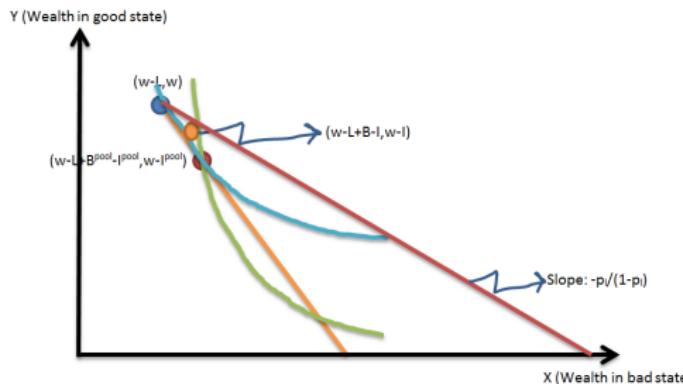
Effective January 1, 2016

| Adjusted Net Income | One Person | Family of Two | Family of Three or More |
|---------------------|------------|---------------|-------------------------|
| \$0 - \$22,000 | \$0.00 | \$0.00 | \$0.00 |
| \$22,001 - \$24,000 | \$12.80 | \$23.20 | \$25.60 |
| \$24,001 - \$26,000 | \$25.60 | \$46.40 | \$51.20 |
| \$26,001 - \$28,000 | \$38.40 | \$69.60 | \$76.80 |
| \$28,001 - \$30,000 | \$51.20 | \$92.80 | \$102.40 |
| Over \$30,000 | \$75.00 | \$136.00 | \$150.00 |

- Relatively inexpensive premium

Affordable Care Act in the U.S.

- ▶ Signed into law in 2010
- ▶ Required insurance companies to offer insurance to patients with pre-existing conditions (i.e., high risk types)
- ▶ Now insurance companies cannot "cream-skim" low risk types



- ▶ Transition from the orange dot to the red dot; because firms have to break even, the price of benefit is likely to rise

Affordable Care Act in the U.S.

SEARCH

The New York Times

POLITICS

Newest Policyholders Under Health Law Are Sicker and Costlier to Insurers

By ROBERT PEAR MARCH 30, 2010

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SEARCH

The New York Times

POLITICS

Newest Policyholders Under Health Law Are Sicker and Costlier to Insurers

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Affordable Care Act in the U.S.

Some of the people buying insurance under the health care law come from states' "high-risk pools," created specifically for people with cancer, heart disease or other serious medical problems.

Affordable Care Act in the U.S.

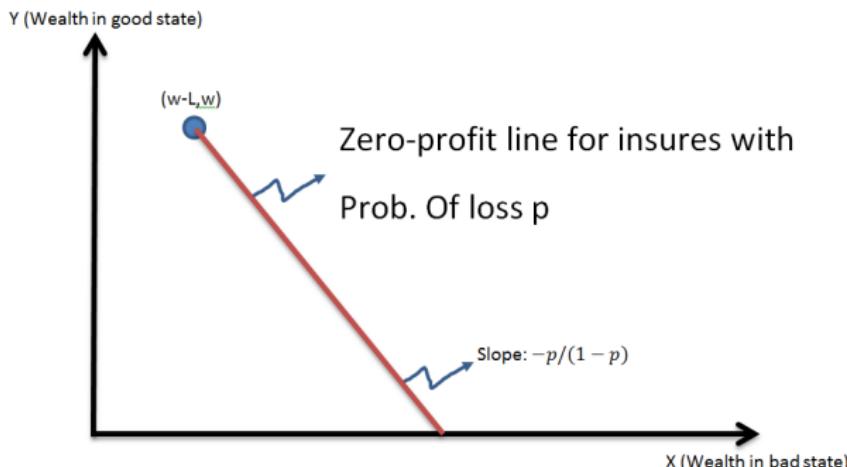
Because insurers' premiums have to cover their medical expenses, the new report helps explain why Blue Cross plans have sought, and insurance commissioners have approved, substantial rate increases in many states. Another round of rate review is about to begin, with insurers generally required to file rate requests for 2017 in the next two months.

Recap of the Lecture on March 31st

- ▶ When there is asymmetric information in the insurance market, there is no pooling equilibrium
- ▶ Why? If there were, then insurance companies would be breaking even with one insurance policy sold to both types
 - ▶ making positive profits on low-risk types
 - ▶ making negative profits on high risk types
- ▶ Low-risk type is said to "cross-subsidize" high-risk type
- ▶ But we were able to find an alternative policy that
 1. Attracts only the low risk types
 2. makes positive profits on low risk types

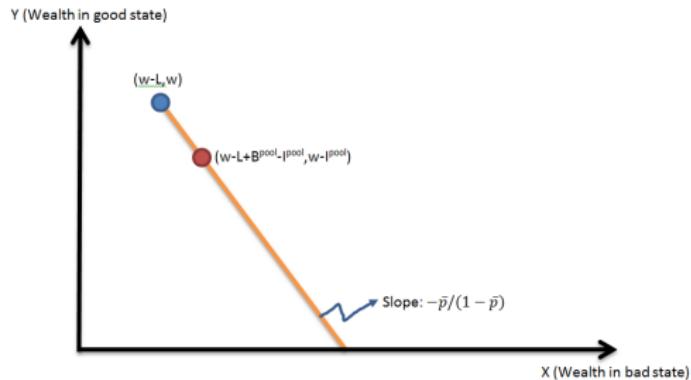
Zero Profit Line

- ▶ A group of consumers with known prob. of loss p
- ▶ On the X-Y plane, we would like to plot the set of policies that would make zero profit if sold to this group

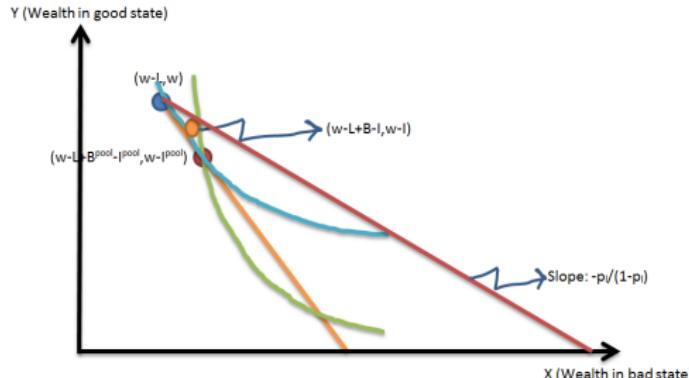


- ▶ Why? Pset 6
- ▶ Insurance policies above the zero profit line would make negative profits
- ▶ Insurance policies below the zero profit line would make positive profits

Candidate for Pooling Equilibrium



Profitable Cream-Skimming of Low Types



- ▶ Which type prefers red dot to orange dot?
- ▶ Would firms make positive profits when only low-risk types buy orange-dot policy?

Separating Equilibrium

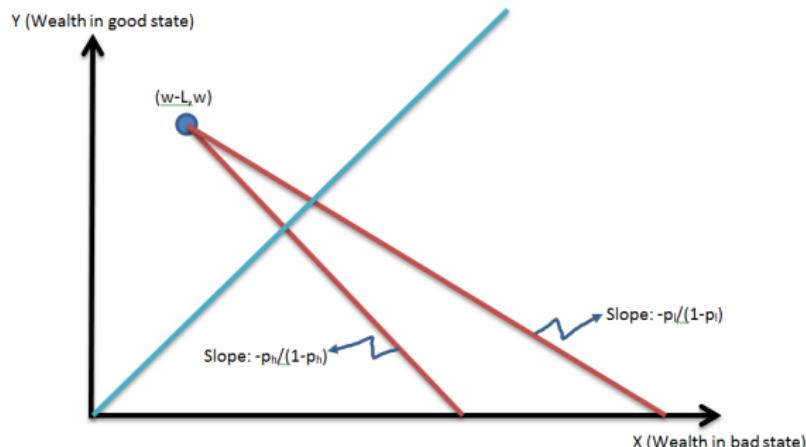
- ▶ Now that we know there are no pooling equilibria, let's try to find separating equilibria where different risk types purchase different policies
- ▶ In a separating equilibrium, there will be two policies offered
 1. A policy meant for high risk types (H-policy)
 2. A policy meant for low risk types (L-policy)
- ▶ But types are unobservable; the issue is how to design H-policies and L-policies so that
 - ▶ High risk types would rather purchase H-policy
 - ▶ Low risk types would rather purchase L-policy
- ▶ How to find such policies?

How to Find the Candidate Policies for Separating Equilibrium

- ▶ We want to find H- and L-policy such that satisfy the following conditions
 - Condition 0. Firms make nonnegative profits on each of H- and L-policy (also equilibrium condition 1)
 - Condition 1. Both types are better off with the insurance than without
 - Condition 2. High risk types prefer H-policy to L-policy
 - Condition 3. Low risk types prefer L-policy to H-policy
 - Condition 4. There are no unoffered policies that would be profitable if offered (also equilibrium condition 2)

Condition 0: Nonnegative Profits from Both Policies

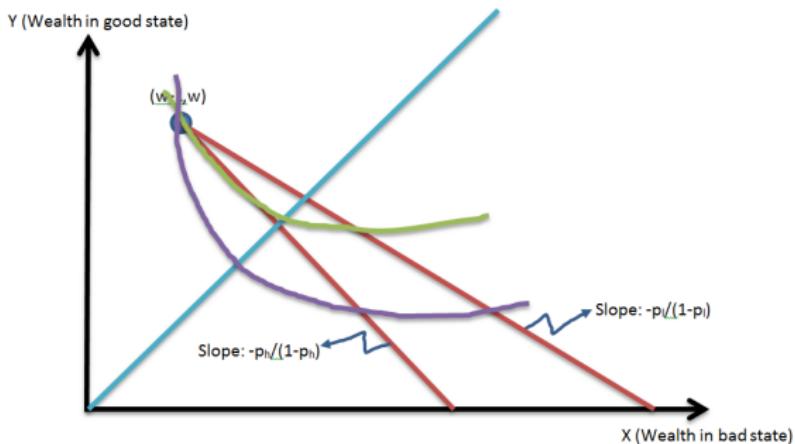
- ▶ Zero profit line for each type



- ▶ To make nonnegative profit, where should the two policies be located?
 - ▶ H-policy should be located on or below...
 - ▶ L-policy should be located on or below...

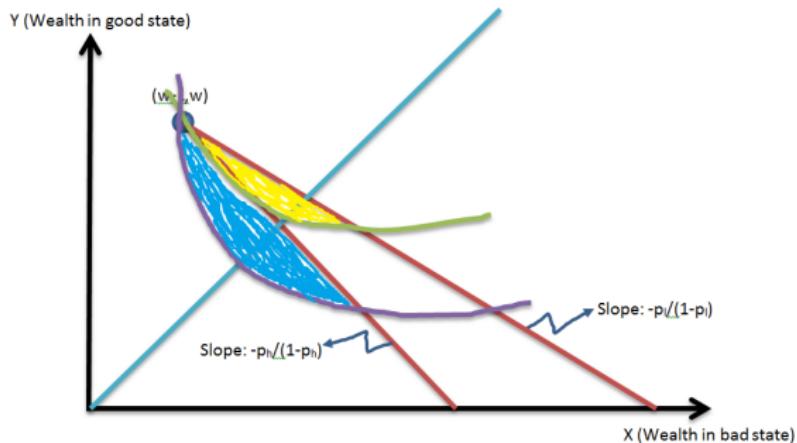
Condition 1: Both Types Are Better Off With Insurance

- ▶ Of course, the potential insurees should be willing to purchase the policy
- ▶ They should be at least as well off with the insurance as without



Condition 2 and 3

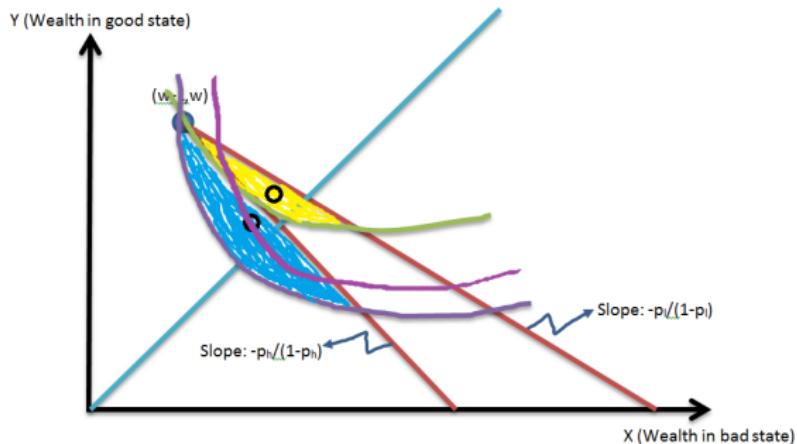
- ▶ High risk types prefer H-policy; Low risk types prefer L-policy



- ▶ Will any one policy in the yellow region and any one policy in the blue region separate the two types?

Condition 2 and 3

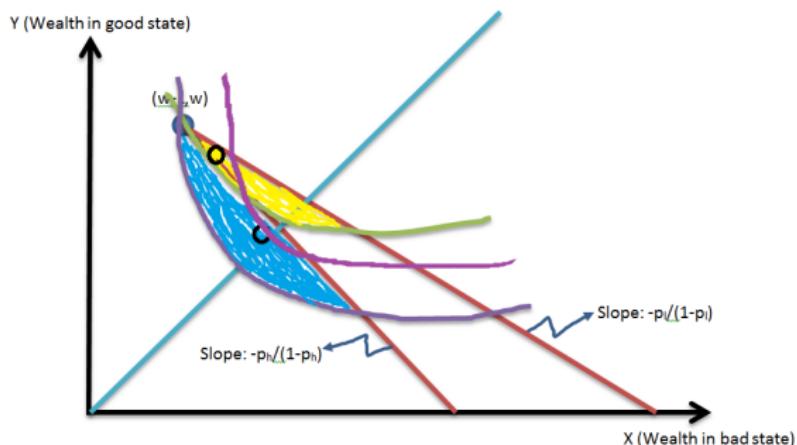
- ▶ What about the following two policies?



- ▶ Why not?

Condition 2 and 3

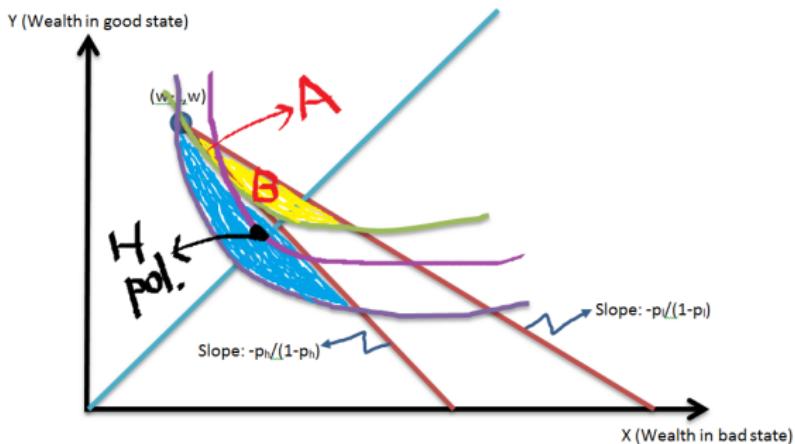
- ▶ What about the following two policies? Will these two policies separate the two types?



- ▶ Yes

Condition 2 and 3

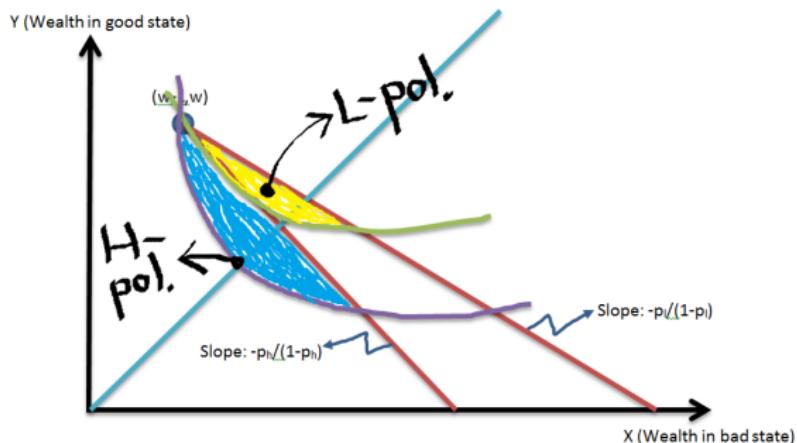
- In general, if the black dot is the H-policy, then which region should the L-policy be? A or B?



- A, because if L-policy is in B, there is no separation
- L-policy should be below the indifference curve of H-type on H-policy

Condition 2 and 3

- ▶ Then can a H-policy be such that H-types are indifferent between it and no insurance?

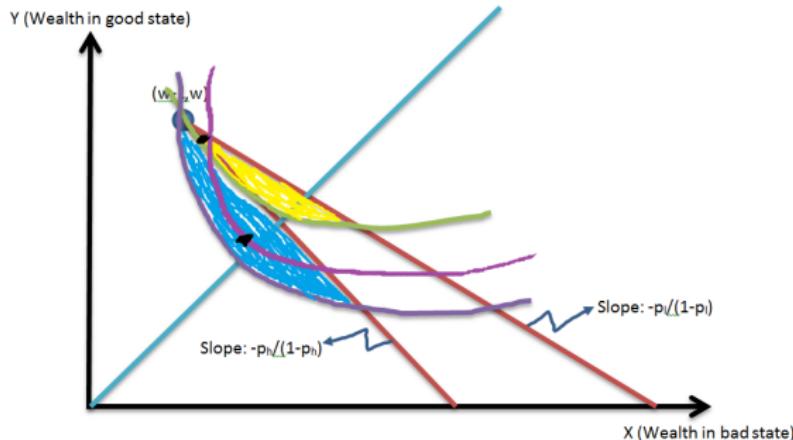


- ▶ No, because then any L-policy in the yellow region will be preferred by H-types, leading to no separation

Condition 2 and 3

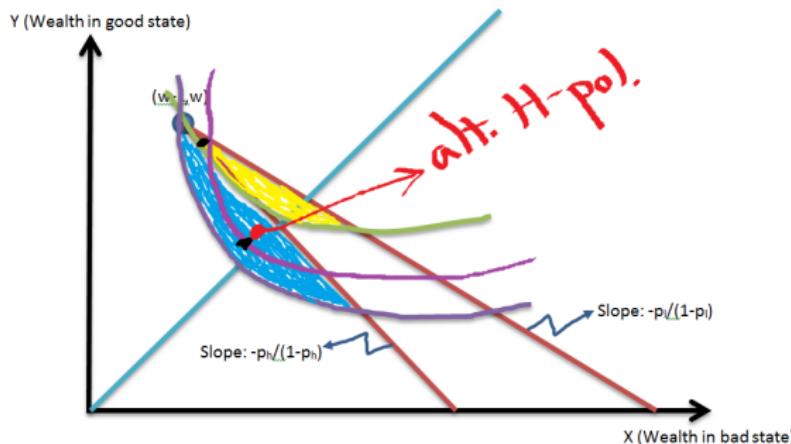
- ▶ So far we learned that, to separate the two types
 - ▶ H-type should be better off with H-policy than without any insurance
 - ▶ L-policy should be under the indiff. curve of H-type on H-pol.
- ▶ So H-policy determines the location of the indifference curve of H-type
- ▶ Then the location of the L-policy is determined accordingly
- ▶ Let's focus on the location of H-policy first

The Location of the Candidate H-policy



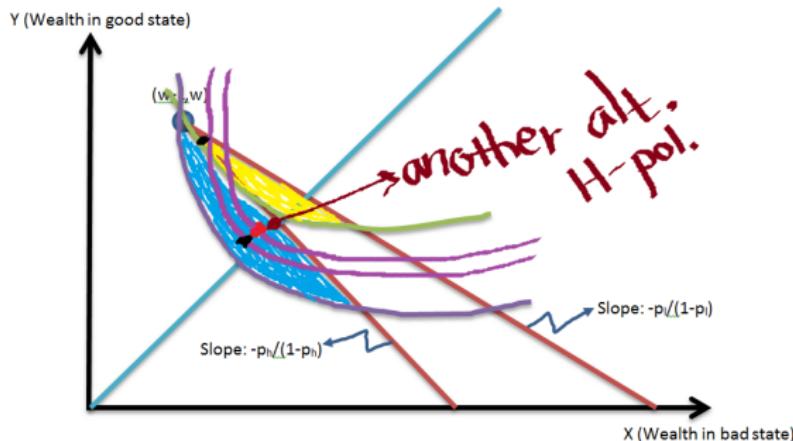
- ▶ Conditions 0 through 3 are satisfied
 - ▶ Positive profits are made at the black dots (why?)
 - ▶ Both types are better off than the endowment point
 - ▶ H types prefer the H policy
 - ▶ L types prefer the L policy
- ▶ Condition 4 (which is also eq. condition 2): any unoffered alternative policy that would be profitable if offered?

The Location of the Candidate H-policy



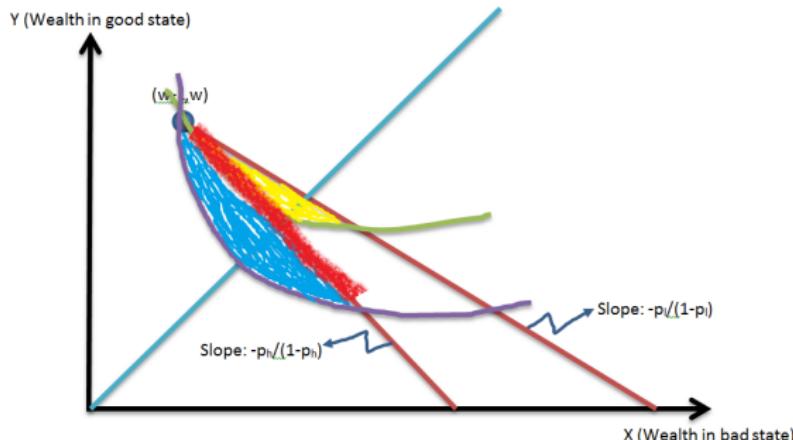
- ▶ Red dot: alternative H-policy, the same L-policy is offered
- ▶ Obviously, conditions 0 through 3 are satisfied
- ▶ Red dot sucks up all the H-type demand of insurance from the black dot and make profits
- ▶ So the black dots cannot be the candidate for separating equilibrium policies

The Location of the Candidate H-policy



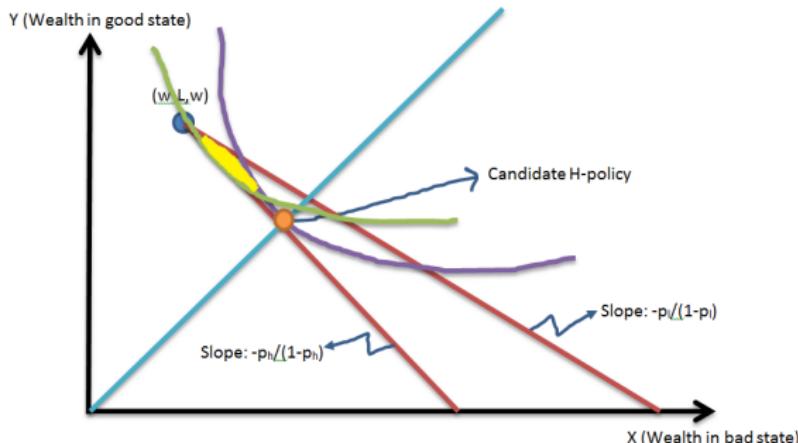
- ▶ Again, if the brown dot is offered as H-policy (and the same L-policy), then no H-type will demand the red dot anymore;
- ▶ So we can infer that the candidate H-policy should be such that it is impossible to offer any policy that would make H-type happier
- ▶ Where would be such H-policy?

The Location of the H-policy



- ▶ Where would H-type be happiest among the policies in the blue region?
- ▶ A good guess would be somewhere on the zero-profit line
- ▶ Among the policies along the zero profit line, which would make H-types happiest?
 - ▶ The full insurance point!

The Location of the Candidate L-policy

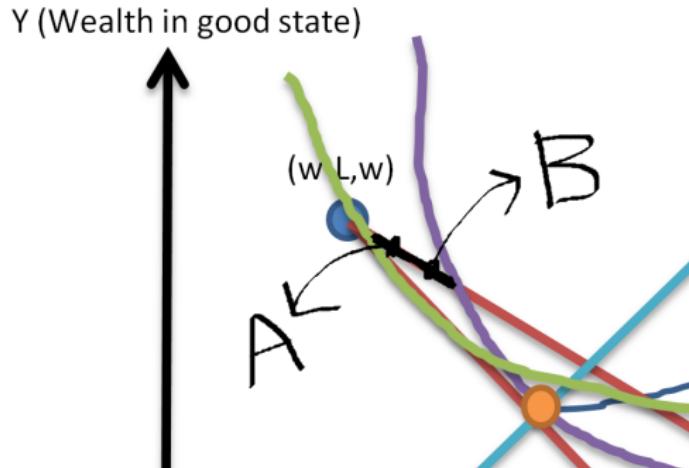


- ▶ Now the candidate H-policy has been located, let's find the candidate L-policy
- ▶ It should be in the yellow region. It is surrounded by
 1. Zero profit line for the L-type (why?)
 2. Indiff. curve of H-types on the candidate H-policy (why?)
 3. Indiff. curve of L-types on the endowment point (why?)

The Location of the Candidate L-policy

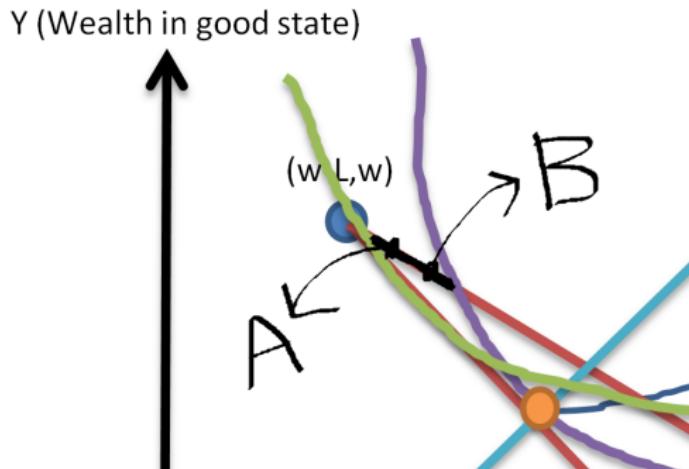
- ▶ Can the candidate L-policy be below the zero profit line for the L-types?
- ▶ No; an alternative L-policy that is "closer" to the zero profit line can steal all the demand
- ▶ Therefore, the candidate L-policy should be ON the zero profit line

The Location of the Candidate L-policy



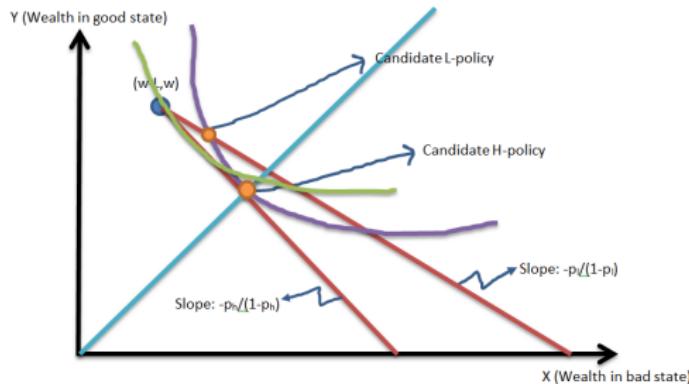
- ▶ The candidate L-policy should be on the black line
- ▶ Suppose A is offered to the L-types
- ▶ If an alternative policy B is offered, will anyone demand A?
 - ▶ No (why?)
- ▶ Will B make (nonnegative) profits?
 - ▶ Yes (why?)

The Location of the Candidate L-policy



- ▶ A is not the candidate L-policy because B, when offered, makes a nonnegative profit, while A gets no demand
- ▶ But B should not be the candidate L-policy either; what should be the candidate policy?
 - ▶ The policy on the black line at which there is no alternative policy that L-types prefer to it
 - ▶ The intersection of the zero profit line AND the H-types' indiff. curve

The Location of the Candidate Policies

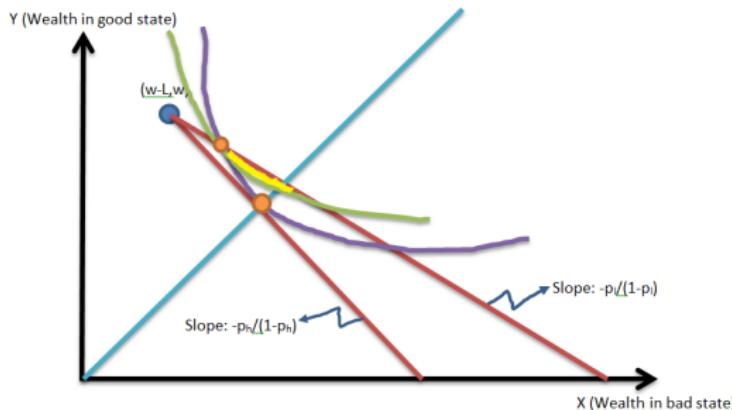


- ▶ Conditions 0 through 3 are satisfied
- ▶ Both policies make zero profits
- ▶ H-types get full insurance; L-types do not
- ▶ H-types indiff. between H- and L-policies; assume H-types get H-policy only

The Candidate for Separating Equilibrium

- ▶ But this is still just a candidate for the separating equilibrium
- ▶ For this to be a separating equilibrium, it has to satisfy the equilibrium conditions
- ▶ Condition 1 satisfied; what about condition 2? (there should not be an unoffered policy that would be profitable)
- ▶ What kind of policy can potentially break the separating equilibrium?
- ▶ Again, there are too many possibilities to try everything; so let's think through this
- ▶ Remember what broke the candidate for the pooling equilibrium was cream-skimming, or separating the low type from the high type
- ▶ What happens when we try to do the same here?

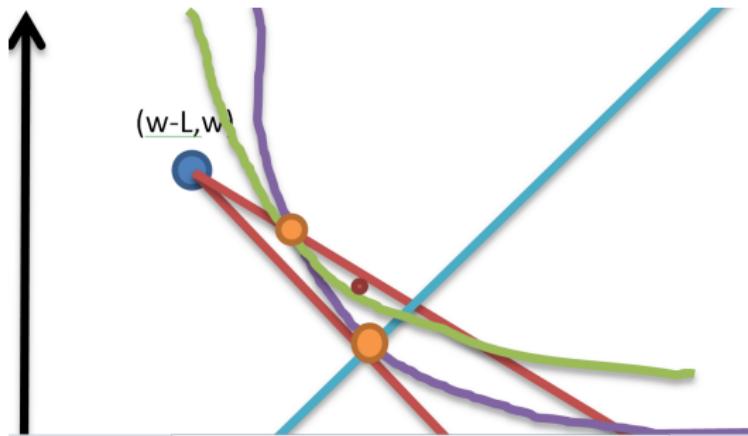
The Candidate for Separating Equilibrium



- ▶ In our attempt to cream-skim low types, we should offer an alternative policy to L-types in the yellow region surrounded by
 - ▶ Zero profit line for L-type (why?)
 - ▶ L-types' indiff. curve on the candidate L-policy (why?)

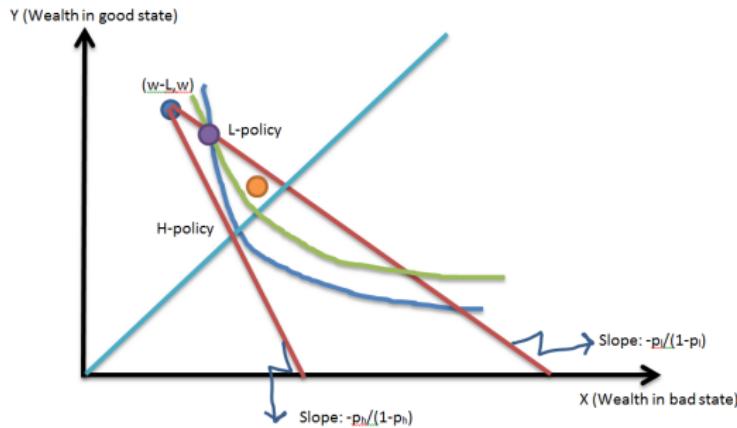
The Candidate for Separating Equilibrium

- ▶ Suppose an alternative L-policy, the small red dot, is offered in addition to the other two policies; will it only attract L-types?



- ▶ So profitable cream-skimming of L-types is impossible
- ▶ Then any potential policy that could break the candidate policies should be attracting both types

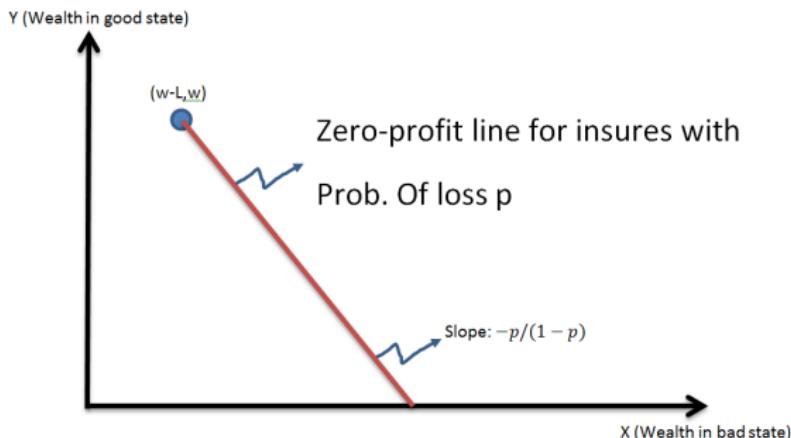
The Candidate for Separating Equilibrium



- ▶ Now we need to determine whether the orange dot makes positive profits if both types purchase it
 - ▶ If so, then H-policy and L-policy do not constitute a separating equilibrium
 - ▶ If no, then they are the separating equilibrium policies
- ▶ All we need to know is whether the orange dot is above/on/below the zero profit line for both types

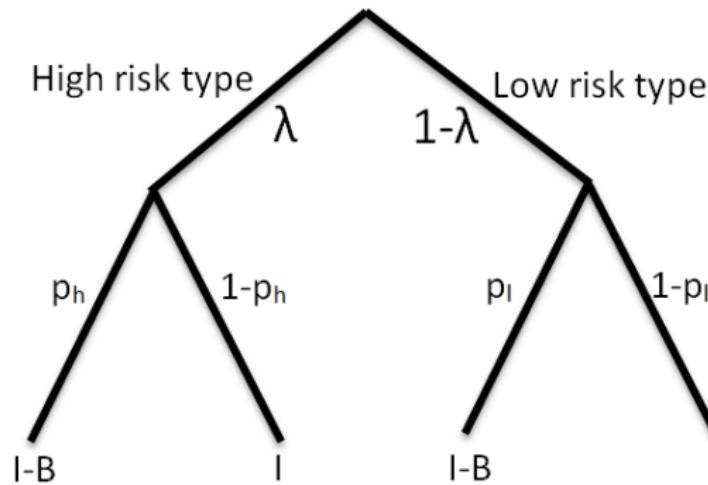
Zero Profit Line When Both Types Buy the Same Policy

- ▶ A group of consumers with known prob. of loss p



- ▶ When a policy is sold to both risk types, what is the prob. of loss?
- ▶ A policy sold to both risk types is like a compound lottery

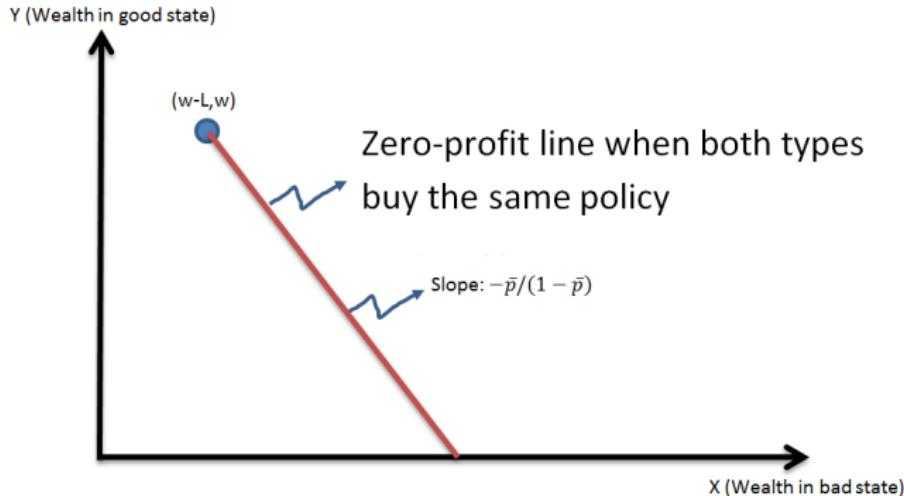
A Policy Sold to Both Risk Types is a Compound Lottery



Probability of loss

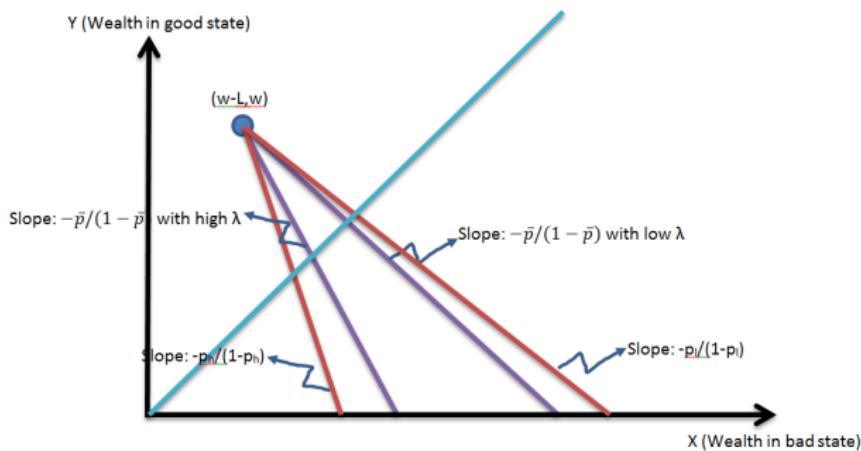
$$\begin{aligned} &= (\text{Prob. an insuree is high risk type}) \times (\text{Prob. loss for high risk type}) \\ &+ (\text{Prob. an insuree is low risk type}) \times (\text{Prob. loss for low risk type}) \\ &= \lambda \times p_h + (1 - \lambda) \times p_l \equiv \bar{p} \end{aligned}$$

Zero Profit Line When Both Types Buy the Same Policy



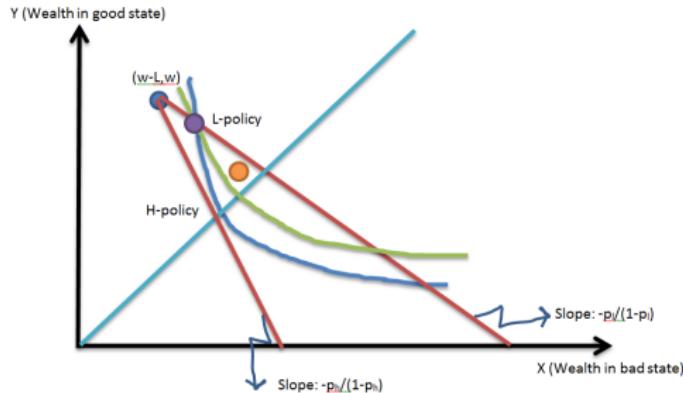
- ▶ Where would the zero profit line for low risk types and high risk types be?
- ▶ How would it be different depending on λ ?

Zero Profit Line When Both Types Buy the Same Policy



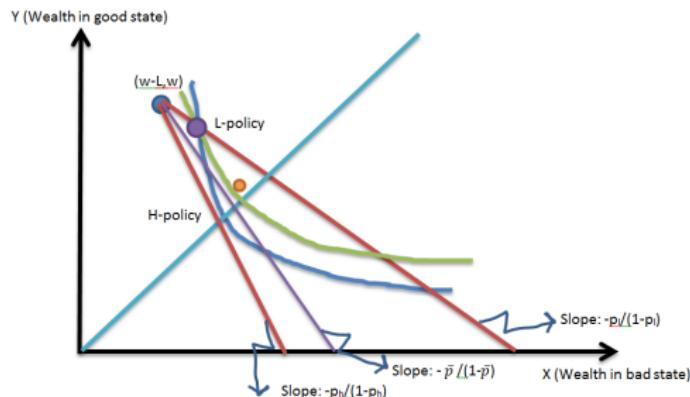
- ▶ With large λ , the zero profit line for pooling policies is closer to the zero profit line for high risk type
- ▶ With small λ , the zero profit line for pooling policies is closer to the zero profit line for low risk type

A Candidate for Separating Equilibrium



- ▶ Will the orange dot be on/below/above the zero profit line?
- ▶ The answer: it depends on whether
 1. The zero profit line for both types
 2. The indiff. curve for the L-types on the L-policy intersect or not

When the L types' Indiff. Curve Does Not Intersect With the Zero Profit Line

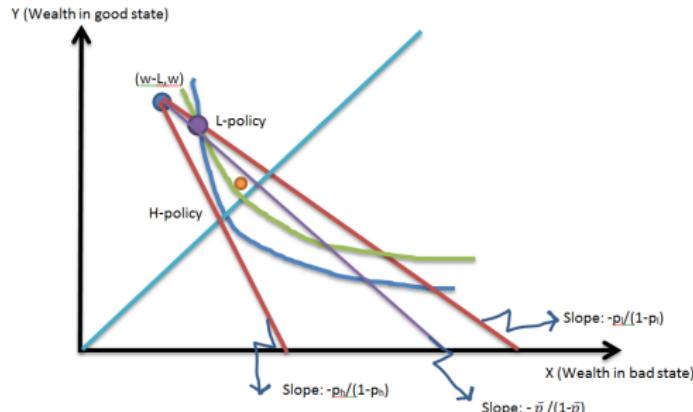


- ▶ This will be the case if λ is large
- ▶ Will the orange dot provide nonnegative profits?
- ▶ It is above the zero-profit (purple) line
- ▶ So no, it will provide negative profits

When the L types' Indiff. Curve Does Not Intersect With the Zero Profit Line

- ▶ When the population is mostly high risk, any pooling policies that attract both types away from the candidate policies make negative profits
- ▶ This means that no alternative policies can break the candidate policies
- ▶ We found an equilibrium at last!!!
- ▶ What is the welfare cost of asymmetric information?
 - ▶ High risk types: no cost; they get full insurance, which they would get without private info.
 - ▶ Low risk types: less than full insurance; bear the entire cost of private info.
- ▶ What about the other case?

A Candidate for Separating Equilibrium

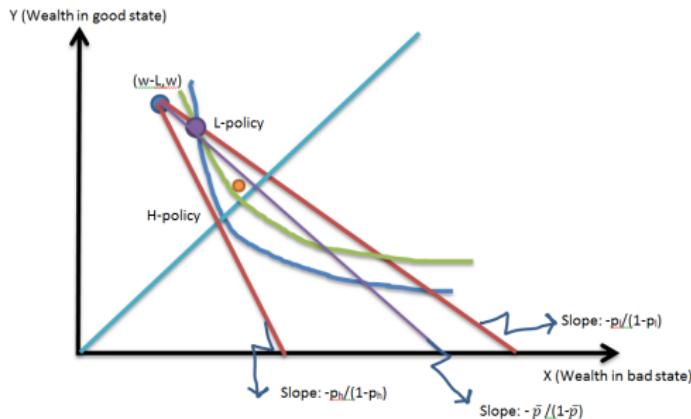


- ▶ This will be the case if λ is small
- ▶ Will the orange dot provide positive profits?
- ▶ It is below the zero-profit line
- ▶ So yes, it will make a positive profit

When the L types' Indiff. Curve Intersects With the Zero Profit Line

- ▶ When the population is mostly low risk, some pooling policies that attract both types away from the candidate policies make positive profits
- ▶ This means that the equilibrium condition 2 is violated
- ▶ The separating equilibrium does not exist in this case
- ▶ Wait, if there are no separating equilibria, and we know that there are no pooling equilibria (regardless of λ), it means that there are no equilibria at all!!!

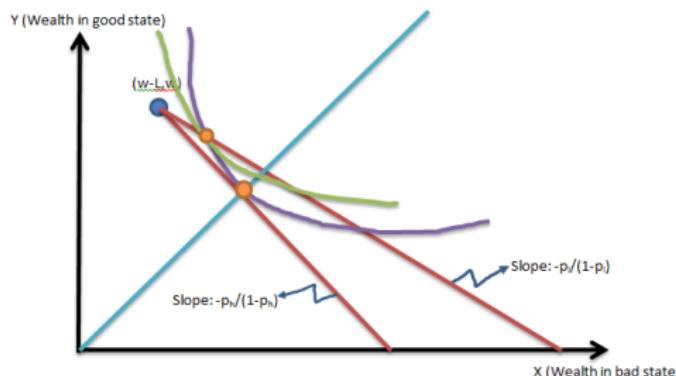
Why Does the Separating Equilibria Fail When λ is Small



- ▶ Because L-policy is on the zero-profit line, the price of a dollar of benefit is actuarially fair
- ▶ But under L-policy, L-types get little insurance
- ▶ L-types are better off with a pooling policy where they get a little more insurance at a worse price
- ▶ In pooling policies, firms suffer losses from H-types; but there are so few of them; so firms make profits as well

Recap of the Lecture on April 5th

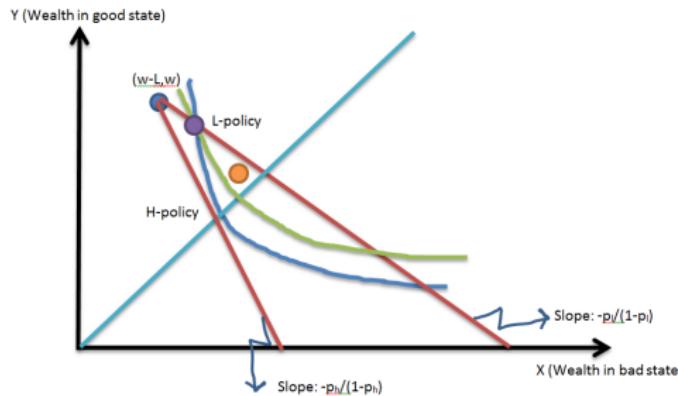
- With pooling equilibrium ruled out, our only hope for an equilibrium in the insurance market with asymmetric information is a separating equilibrium
- The candidate for a separating equilibrium



- Satisfy the following conditions
 - Nonnegative profits from each policy
 - Both types are better off with the insurance than without
 - H-type prefers H-policy; L-type prefers L-policy

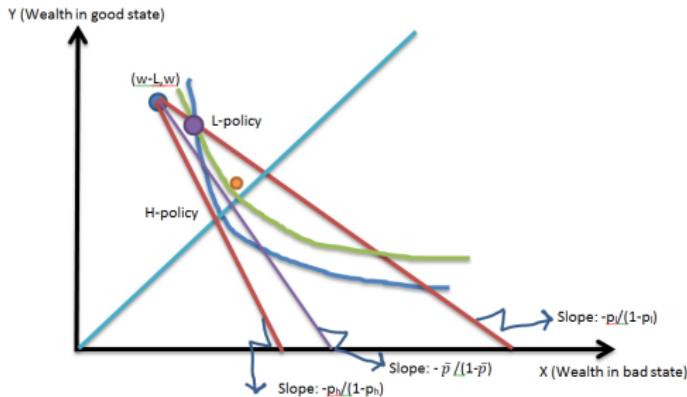
Recap of the Lecture on April 5th

- ▶ This candidate may not actually be the separating equilibrium if there is an alternative policy that is profitable if offered



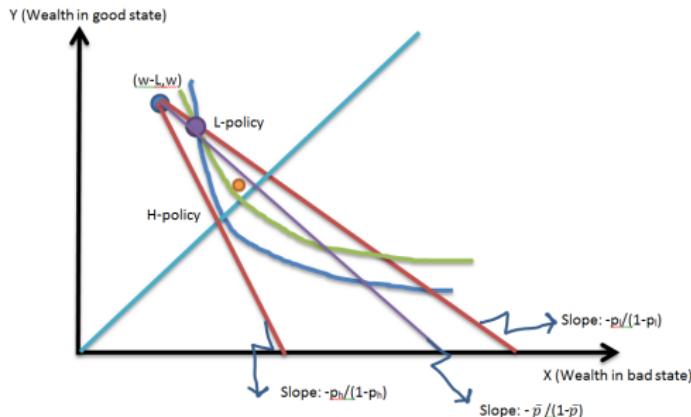
- ▶ An alternative as above might be profitable
- ▶ It depends on the proportion of high risk type in the population

Recap of the Lecture on April 5th



- ▶ In this case, the zero profit line for pooling policies is close to the zero profit line for H -type because λ is large
- ▶ The alternative is not profitable; so the candidate is indeed the separating equilibrium
- ▶ Welfare loss due to asymmetric information: L -type gets less than full insurance

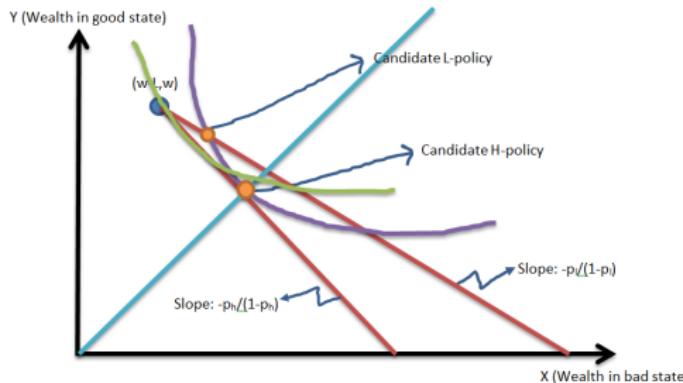
Recap of the Lecture on April 5th



- ▶ In this case, the zero profit line for pooling policies is close to the zero profit line for L-type because λ is small
- ▶ The alternative is profitable; so the candidate is not an equilibrium (for it violates eq. condition 2)
- ▶ No pooling eq.; no separating eq. \Rightarrow no equilibrium

The Implication of the Equilibrium When It Exists

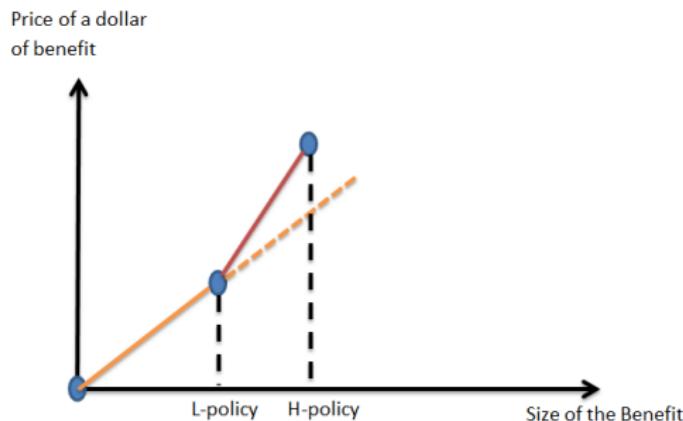
- ▶ This model generates a few predictions that we can test with data
- ▶ Look at the equilibrium policies



- ▶ The relationship between the size of the benefit and the price of a dollar of benefit
- ▶ The relationship between risk and the amount of benefit purchase

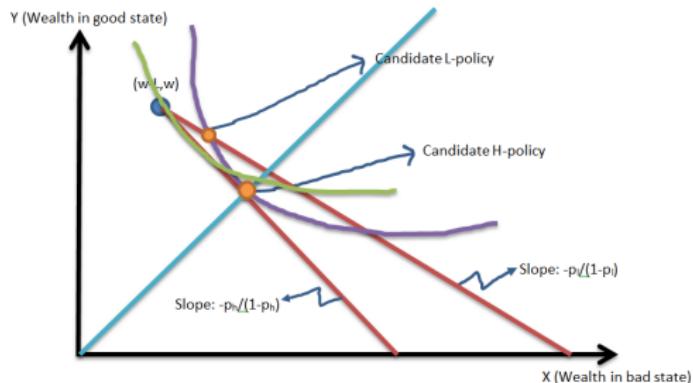
The Relationship Between the Price and the Quantity

- ▶ We can plot how the price of a dollar of benefit changes as the size of the benefit increases



- ▶ Price increases nonlinearly as the benefit increases
- ▶ Why? If the price per dollar of benefit for H-policy is the same as L-policy, firms will not be able to break even on H-policies (H-policy would be above the zero-profit line for H-types)
- ▶ Therefore, per dollar price of H-policy has to be more expensive than that of L-policy

The Relationship Between the Risk and the Quantity



- ▶ High risk types get a larger coverage at a higher price per dollar of benefit
- ▶ Low risk types get a smaller coverage at a lower price per dollar of benefit

Testing the Model with Life Insurance Data

- ▶ Based on Cawley and Philipson's 1999 article published in the American Economic Review, titled *An Empirical Examination of Information Barriers to Trade in Insurance*
- ▶ Why is life insurance market a good market to study?
 - ▶ Largest private individual insurance market
 - ▶ 58% of the premia paid to any type of insurance was for life insurance
 - ▶ 3.6 percent of the U.S. GDP; most widely held financial product (more than savings account)
 - ▶ Prime example of market that suffers inefficiency due to asymmetric information
 - ▶ People die of all kinds of reasons (accident, suicide, undetected disease); cost of measuring the level of risk of every potential insuree might be too high
 - ▶ But insurees might have private information about their likelihood of death (accident-proneness; depression/disease without getting treatment)

Prediction 1: Does the Unit Price Rise as Coverage Increases?

- ▶ A random sample of 28,000 policies issued in the U.S. in 1994

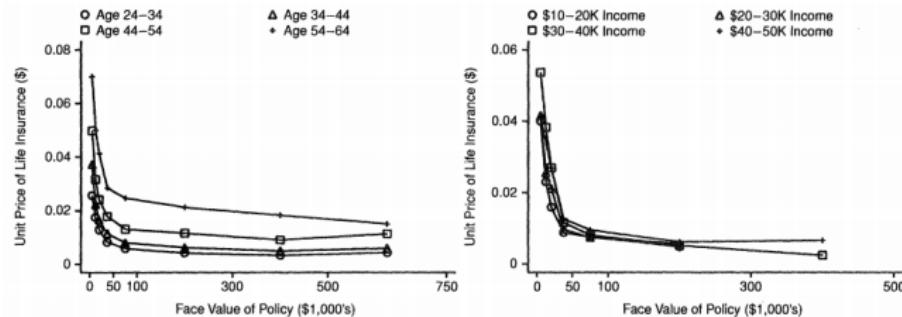


FIGURE 4. LIMRA UNIT PRICES: (A) UNIT PRICE BY AGE OF INSURED; (B) UNIT PRICE BY INCOME OF INSURED

Note: Unit price is the cost per dollar of coverage.

- ▶ Price falls as the coverage increases, contrary to the prediction of the model
- ▶ This data can sort policies only based on the age of the insureds and income; would the conclusion continue to hold with more disaggregated data?

Prediction 1: Does the Unit Price Rise as Coverage Increases?

TABLE 1—UNIT PRICE AND QUANTITY: MEAN CHANGE IN UNIT PRICE ACROSS FIRMS IN COMPU LIFE

| Risk group | Change in policy size | | | | |
|---|-----------------------|------------------|------------------|------------------|------------------|
| | \$100–500K | \$500–1,000K | \$1,000–1,500K | \$1,500–2,000K | \$2,000–2,500K |
| 30-year-old nonsmoking preferred females | −49.90 (−25.86) | −7.88 (−5.27) | −1.96 (−1.59) | −0.98 (−0.68) | −0.59 (−0.38) |
| 30-year-old nonsmoking nonpreferred females | −46.18 (−21.63) | −8.03 (−4.69) | −1.76 (−0.91) | −0.88 (−0.45) | −0.53 (−0.27) |
| 30-year-old smoking preferred females | −45.00 (−7.65) | −5.25 (−0.90) | −1.81 (−0.30) | −0.90 (−0.15) | −0.87 (−0.15) |
| 30-year-old smoking nonpreferred females | −51.69 (−13.18) | −6.63 (−1.44) | −1.43 (−0.30) | −0.72 (−0.15) | −0.43 (−0.09) |
| 30-year-old nonsmoking preferred males | −51.03 (−12.42) | −8.22 (−2.45) | −1.76 (−0.76) | −0.91 (−0.44) | −0.55 (−0.28) |
| 30-year-old nonsmoking nonpreferred males | −46.08 (−16.90) | −8.07 (−2.89) | −1.65 (−0.55) | −0.83 (−0.27) | −0.50 (−0.16) |
| 30-year-old smoking preferred males | −57.83 (−15.60) | −6.92 (−1.62) | −1.47 (−0.39) | −0.74 (−0.19) | −0.44 (−0.11) |
| 30-year-old smoking nonpreferred males | −57.02 (−9.14) | −6.46 (−1.10) | −1.47 (−0.25) | −0.73 (−0.13) | −0.44 (−0.08) |

- ▶ The conclusion does not change; price per 100,000 dollar of benefit FALLS as the coverage increases

Prediction 2: Do High Risk Types Buy More Insurance?

- ▶ There can be two margins that we can look at
 - ▶ Extensive margin: high risk types hold life insurance more than low risk types
 - ▶ Intensive margin: among the life insurance holders, high risk types hold life insurance with higher coverage than low risk types

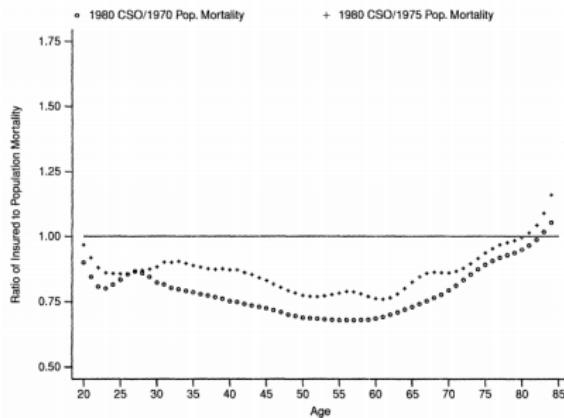


FIGURE 2. MORTALITY OF MALES, 1970–1975

- ▶ Those with life insurance are less likely to die than those without; low risk types hold life insurance more

Prediction 2: Do High Risk Types Buy More Insurance?

- ▶ To see if there is any relationship between the risk and the size of the benefit in the intensive margin, the authors ran a regression

$$\begin{aligned}\log(\text{benefit}) = & \beta_1(\text{self-perceived risk}) + \beta_2(\text{wealth}) \\ & + \beta_3(\text{loss of income insured}) \\ & + \beta_4(\text{number of grandchildren}) \\ & + \beta_5(\text{number of children}) + \dots\end{aligned}$$

- ▶ Self-perceived risk: probability of death in the next 11 to 15 years felt by the survey respondents
- ▶ If the prediction is correct, what should be the sign of β_1 ?

Prediction 2: Do High Risk Types Buy More Insurance?

TABLE 6—EQUILIBRIUM DEMAND AND SELF-PERCEIVED RISK IN THE HRS AND AHEAD DATA
(DEPENDENT VARIABLE: LOG OF TERM INSURANCE AWARD)

| Self-perceived risk (SPR) | HRS | | | | | | AHEAD | | | |
|------------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | Age 51–54 | | Age 55–58 | | Age 59–61 | | Group A | | Group B | |
| | Model 1 | Model 2 |
| 0.0 < SPR ≤ 0.1 | 0.27 (1.60) | 0.28 (1.69) | 0.53 (3.39) | 0.52 (3.41) | 0.18 (0.96) | 0.20 (1.02) | 0.23 (0.88) | 0.11 (0.47) | -0.48 (-1.52) | -0.37 (-1.26) |
| 0.1 < SPR ≤ 0.2 | 0.21 (1.82) | 0.23 (2.05) | 0.29 (2.58) | 0.30 (2.71) | 0.04 (0.26) | 0.07 (0.47) | -0.08 (-0.38) | 0.02 (0.13) | 0.21 (0.76) | 0.19 (0.72) |
| 0.2 < SPR ≤ 0.3 | 0.31 (2.38) | 0.28 (2.30) | 0.08 (0.58) | 0.07 (0.55) | 0.16 (0.94) | 0.14 (0.82) | -0.04 (-0.20) | -0.02 (-0.12) | 0.17 (0.66) | 0.20 (0.84) |
| 0.3 < SPR ≤ 0.4 | 0.04 (0.22) | 0.02 (0.11) | 0.05 (0.29) | 0.08 (0.51) | -0.22 (-0.94) | -0.22 (-0.94) | -0.13 (-0.35) | -0.04 (-0.12) | 0.33 (0.70) | 0.18 (0.35) |
| 0.4 < SPR ≤ 0.5 | -0.10 (-0.95) | -0.08 (-0.77) | -0.05 (-0.77) | -0.06 (-0.39) | 0.08 (-0.54) | 0.08 (0.61) | 0.07 (0.53) | -0.14 (-1.14) | -0.16 (-1.47) | 0.18 (0.94) |
| 0.5 < SPR ≤ 0.6 | 0.02 (0.10) | 0.01 (0.02) | 0.18 (1.17) | 0.16 (0.99) | 0.26 (1.10) | 0.27 (1.16) | -0.14 (-0.54) | -0.22 (-0.92) | -0.09 (-0.30) | -0.15 (-0.46) |
| 0.6 < SPR ≤ 0.7 | 0.08 (0.44) | 0.06 (0.34) | -0.27 (-1.48) | -0.30 (-1.68) | -0.29 (-1.18) | -0.36 (-1.49) | 0.08 (0.39) | -0.17 (-0.86) | 0.18 (0.58) | 0.13 (0.44) |
| 0.7 < SPR ≤ 0.8 | 0.18 (0.83) | 0.09 (0.42) | -0.25 (-1.32) | -0.20 (-1.06) | 0.09 (0.40) | 0.09 (0.43) | -0.08 (-0.45) | -0.13 (0.80) | 0.27 (1.17) | 0.25 (1.25) |
| 0.8 < SPR ≤ 0.9 | -0.14 (-0.41) | -0.12 (-0.36) | 0.07 (0.25) | -0.01 (-0.05) | -0.91 (-3.34) | -0.91 (-3.40) | 0.00 (0.00) | -0.07 (-0.37) | 0.19 (0.85) | 0.22 (1.06) |
| 0.9 < SPR ≤ 1.0 | -0.31 (-1.62) | -0.40 (-2.08) | -0.15 (-0.88) | -0.17 (-0.99) | -0.10 (-0.45) | -0.12 (-0.55) | -0.23 (-1.83) | -0.18 (-1.49) | 0.07 (0.36) | 0.11 (0.65) |

- Contrary to the prediction of the model, β_1 not significantly different from 0, except for the circled in red

Why Are the Predictions Wrong?

- ▶ Author's explanation
 - ▶ At least in the life insurance market, there might be little meaningful private information
 - ▶ Why? The cost of "underwriting", where the insurance company assesses the risk of life insurance applicants, might not be too high
 - ▶ Insurers ask the applicants to fill out an application and a medical questionnaire
 - ▶ Underwriting is likely to eliminate asymmetric information between the insurer and the insuree
 - ▶ Therefore, the asymmetric information might not be very relevant to the life insurance market

U.K. Market for Annuities

- ▶ Is there any market where asymmetric information matters?
- ▶ According to an article by Finklestein and Poterba published in 2004, yes
- ▶ The authors study the U.K. market for annuities
- ▶ What is annuity?
 - ▶ Insuree pays the insurer premium in the form of lump-sum or a series of regular payments
 - ▶ At a pre-specified time (e.g., when the insuree turns 65), the insurer pays the insuree a pre-specified amount until the insuree dies
 - ▶ Insures the risk of outliving your wealth

U.K. Market for Annuities

- ▶ Other authors have tested the model by looking at the correlation between
 - ▶ Coverage/benefit
 - ▶ Risk
- ▶ The prediction of the model was high risk type gets...
- ▶ Finklestein and Poterba looks at three dimensions of annuities
 1. The yearly benefit one receives in the first year
 2. The degree of backloading
 3. Whether the annuity is guaranteed or not
- ▶ Let's think about the predictions of the model for each of these features

Predictions for Different Features of Annuities

- ▶ Obviously, the model predicts that higher risk types (in this case, the risk is living too long) should get a larger coverage, so the yearly benefit should be larger for them
- ▶ The degree of backloading
 - ▶ There are three types of annuities
 1. One that pays fixed amount each year
 2. One that is inflation adjusted, pays fixed amount in real terms
 3. One where the benefit rises each year
 - ▶ Due to inflation, the purchasing power for 1 declines in later years
 - ▶ If you think you are high risk types, which one would you prefer?

Predictions for Different Features of Annuities

- ▶ Whether the annuity is guaranteed or not
 - ▶ Some annuities come with guaranteed benefit
 - ▶ For example, annuities with 10 year guarantee pays the benefit even if the beneficiary dies before 10 year
 - ▶ After-death benefit accrues to the beneficiaries estate
 - ▶ Why would you buy a guaranteed annuity as opposed to non-guaranteed annuity?
 - ▶ If you believe you are going to die soon, non-guaranteed annuity would be a bad deal
 - ▶ So low risk types are more likely to buy guaranteed annuities

Predictions for Different Features of Annuities

- ▶ Regression

Prob. Death of Insuree i

$$\begin{aligned} &= \beta_1 \times (1 \text{ if } i\text{'s annuity is inflation-adjusted}) \\ &\quad + \beta_2 (1 \text{ if benefit increases each year}) \\ &\quad + \beta_3 (1 \text{ if guaranteed}) + \beta_4 (\text{first year benefit}) \\ &\quad + \beta_5 (1 \text{ if male}) \end{aligned}$$

- ▶ If the prediction is correct, then what should be the sign of $\beta_1, \beta_2, \beta_3, \beta_4$?
 1. $\beta_1 < 0$
 2. $\beta_2 < 0$
 3. $\beta_3 > 0$
 4. $\beta_4 < 0$

Results

TABLE 2
SELECTION EFFECTS AND ANNUITY PRODUCT CHARACTERISTICS

| EXPLANATORY VARIABLE | ESTIMATES FROM HAZARD MODEL OF MORTALITY AFTER PURCHASING AN ANNUITY | | ESTIMATES FROM LINEAR PROBABILITY MODEL OF PROBABILITY OF DYING WITHIN FIVE YEARS | |
|----------------------------|--|-------------------------|---|-------------------------|
| | Compulsory Market (1) | Voluntary Market (2) | Compulsory Market (3) | Voluntary Market (4) |
| Index-linked | -.839*** (.217) | -.894** (.358) | -.053*** (.019) | -.185*** (.050) |
| Escalating | -1.085*** (.113) | -1.497*** (.253) | -.072*** (.010) | -.152*** (.030) |
| Guaranteed | .019 (.029) | .216*** (.060) | .007* (.004) | .046*** (.016) |
| Capital-protected | ... (.051) | .056 (.051) | ... (.016) | .064*** (.016) |
| Payment (£100s) | -.003*** (.0006) | .001** (.0004) | -.0003*** (.0001) | .0003*** (.0001) |
| Male Annuitant | .640*** (.039) | .252*** (.051) | .044*** (.005) | .044*** (.014) |
| Observations | 38,362 | 3,692 | 24,481 | 3,575 |
| Number of deaths in sample | 6,311 | 1,944 | 2,693 | 822 |

- ▶ The prediction of the theory is correct for the degree of backloading and guarantee; not so much for the size of the benefit