

Prepare brief and precise answers to the following questions. You are encouraged to discuss the solutions in groups but should write up the solutions independently.

1. Let  $C_t$  be consumption and  $X_t$  be a predictor of consumption. Suppose you have quarterly data on  $C$  and  $X$ . Let  $D_{1t}$ ,  $D_{2t}$ ,  $D_{3t}$ , and  $D_{4t}$  be dummy variables such that  $D_{1t}$  takes the value 1 in quarter 1 and 0 otherwise,  $D_{2t}$  takes the value 1 in quarter 2 and 0 otherwise, etc. Which of the following, if any, suffer from perfect multicollinearity and why?

- a)  $C_t = \alpha + \beta X_t + \gamma_1 X_t D_{1t} + \gamma_2 X_t D_{2t} + \gamma_3 X_t D_{3t} + \gamma_4 X_t D_{4t} + u_t$
- b)  $C_t = \alpha + \beta X_t + \gamma_1 X_t D_{1t} + \gamma_2 X_t D_{2t} + \gamma_3 X_t D_{3t} + \gamma_4 X_t (1 - D_{1t} - D_{2t} - D_{3t}) + u_t$
- c)  $C_t = \alpha + \delta_1 D_{1t} + \delta_2 D_{2t} + \gamma_1 X_t D_{1t} + \gamma_2 X_t D_{2t} + \gamma_3 X_t D_{3t} + \gamma_4 X_t D_{4t} + u_t$
- d)  $C_t = \alpha + \delta_1 D_{1t} + \delta_2 D_{2t} + \beta X_t + \gamma_1 X_t D_{1t} + \gamma_2 X_t D_{2t} + \gamma_3 X_t D_{3t} + u_t$

(There's nothing special about labeling the parameters with  $\alpha$ ,  $\beta$ ,  $\gamma_1$ ,  $\gamma_2$ ,  $\delta_1$ ,  $\delta_2$ , etc. We could have labeled them  $\beta_1$ ,  $\beta_2$ , ... without changing their interpretation.)

2. a) In models (a)–(d) of question 1, what are the slope coefficients of  $X_t$  in each of the 4 quarters?

b) Suppose you estimate model (c) and wrote down the estimated slope coefficients for  $X_t$  in each of the 4 quarters. You then estimate model (d) and write down the estimated slope coefficients for  $X_t$  in each of the 4 quarters. Do your estimates change? Why or why not?

3. Determine whether the following are true or false and explain why:

- a) Adjusted  $R^2$  can be negative.
- b) Adjusted  $R^2$  can be larger than 1.

4. For this question you will need to load the data file `incomedata.Rda` (available from the NYU classes page) using the command `load("incomedata.Rda")`. Make sure you have the data file stored in your working directory. You will also need to load the package `AER` using the command `require("AER")`.

The data file contains data on earnings (EARN), education (ED), and gender (GEN) for 1192 individuals in the early 1990s.

a) Estimate the regression model:

$$\text{EARN}_i = \beta_0 + \beta_1 \text{GEN}_i + \beta_2 \text{ED}_i + \beta_3 \text{GEN}_i \text{ED}_i + u_i$$

and interpret the estimated coefficients and the adjusted  $R^2$ .

b) Construct 95% confidence intervals for  $\beta_1$  and  $\beta_3$ .

c) Do a  $t$ -test for  $H_0 : \beta_3 = 0$  against  $H_1 : \beta_3 > 0$  at the 5% level of significance.

d) Do a Wald test for  $H_0 : \beta_1 = \beta_3 = 0$ .

e) Now estimate the regression model:

$$\text{LOGINC}_i = \beta_0 + \beta_1 \text{GEN}_i + \beta_2 \text{ED}_i + \beta_3 \text{GEN}_i \text{ED}_i + u_i$$

where  $\text{LOGINC}$  is now the log of the earnings of person  $i$ . How does your answer to (c) change?