Econ 222
Macroeconomic Theory I
Winter Term 2016

Assignment #3
DUE: Drop Box 2nd floor Dunning Hall by NOON on **March 11**
No late submissions, No group submissions, No "photocopy" answers will be accepted

Remarks: Write clearly and concisely. Devote some time to give the graphs, plots and tables a format easy to understand. Also the way you present your answers matter for the final grade. Even if a question is mainly analytical, briefly explain what you are doing, stressing the economic meaning of the various steps. Being able to convey your thoughts effectively is an asset also in real life.

Question 1: The Solow growth model

Consider the following production function:

$$Y(t) = F(K(t), A(t)N(t)), \tag{1}$$

where $Y(t)$ denotes output, $K(t)$ denotes total capital input, $A(t)$ denotes "knowledge", and $N(t)$ denotes total labour input. $F(\cdot,\cdot)$ is the aggregate production function. Note that $A(t)N(t)$ is commonly referred as *effective labour*, and technological progress that enters the production function in this fashion is known as *labour-augmenting* or *Harrod-neutral*.

a) Derive the intense form (per unit of effective labour) of the production function, using the abstract production function above.

b) We know the actual production function is

$$Y(t) = K(t)^{\alpha} \left[A(t)N(t)\right]^{1-\alpha}. \tag{2}$$

Denote the population growth rate as n, the depreciation rate as d, the growth rate of knowledge as g, and the saving rate as s. First derive the steady state condition for this economy (involving investment per unit of effective labour), and then solve for the steady state level of capital per unit of effective labour, $k^*(t)$.

c) Given the result in b), solve for the steady state level of output per worker $y^*(t)$, and consumption per worker $c^*(t)$.

d) Solve for the Golden rule level of capital per worker, k_G. If the government can choose a saving rate for the economy, what saving rate should the government choose, assuming that it wants to achieve k_G in the steady state.

e) Suppose the economy is at a steady state, and there is a decrease in the saving rate s. Explain the impact of this shock regarding steady state variables, *with the help of a diagram*. Label your graph appropriately.

Question 2: *Learning-by-doing* and endogenous growth

This question explores the idea of *learning-by-doing* as a source of economic growth. Consider the same production function as in Question 1:

$$Y(t) = K(t)^{\alpha} \left[A(t)N(t)\right]^{1-\alpha}, \qquad \text{(2 revisited)}$$

where, as before, $Y(t)$ denotes output, $K(t)$ denotes total capital input, and $N(t)$ denotes total labour input. Capital depreciates at a rate d. The key deviation from Question 1 regarding the production side of the economy is that "knowledge" $A(t)$ is now proportional to the capital stock, in particular,

$$A(t) = BK(t), \qquad (3)$$

where B is a positive coefficient. This specification is designed to capture the idea of *learning-by-doing*: a greater capital stock increases the scale of production and the *experience* (of workers, managers, etc) accumulated with production processes, making the production process itself more productive.

a) Substitute Equation 3 into Equation 2 and verify the similarity of this model to the textbook *AK* model.

b) Calculate the growth rate of the capital stock $K(t)$, assuming the goods market is in equilibrium. [Hint: Use the relationship between net and gross investment in Chapter 4.]

c) Define the answer you get in part b) as g_K and assume there is no population growth (i.e. $N(t) = N$), find the growth rate of total output $Y(t)$ in terms of g_K.

d) Using a diagram similar to Figure 6.3 in the textbook (7^{th} edition), explain why this economy can generate unlimited growth.

Question 3: Asset market

Consider the following nominal money demand function:

$$M^d = P \cdot L(Y, r + \pi^e) = P \cdot [100 + 0.8Y - 200(r + \pi^e)], \tag{4}$$

where M^d is the nominal money balances demanded, P is the price level, Y is the total output, r is the real interest rate, and π^e is the expected inflation rate.

(a) Briefly state the *Walras' Law* in the context of the asset market equilibrium.

(b) Assuming that $Y = 500$, $P = 3$, $r = 0.5\%$, and $\pi^e = 0.5\%$, and that the asset market is currently in equilibrium (at $r = 0.5\%$), calculate the nominal money supply M^s for this economy and find the corresponding velocity of money (consider the *quantity theory of money* model).

(c) Find the real income elasticity of money demand at the equilibrium level of money balances you get in (b).

(d) Assuming that the real income is expected to decline by 1.0% over the next year, and the central bank is targeting a 1% inflation rate next year, by how much would you advise the central bank to change the money supply?