MATH233 Unit 4: Logarithmic Functions

Discussion Board Assignment: Version 2A

Note: All work must be shown and explained to receive full credit.

The equation to simulate the time that it will take for an investment to grow based on the interest rate, compounded monthly, is as follows:

$$T(r) = \frac{\ln(k)}{12\ln(1+0.0833r)} \qquad r > 0$$

where T is the time in years, r is the interest rate (in decimal form), and k represents the growth of the investment.

1. Choose a value for k from the table based on the first letter of your last name for the investment that you want to simulate, and set up your equation for $T(r)$. Choose a value of k that is different from your classmates' values of k.

First letter of your last name	**Values for *k***	**Interpretation**
A–F	1.10–1.90	Investment will grow 1.10 to 1.90 times
G–L	2.10–2.90	Investment will grow 2.10 to 2.90 times
M–R	3.10–3.90	Investment will grow 3.10 to 3.90 times
S–Z	4.10–4.90	Investment will grow 4.10 to 4.90 times

2. Find the derivative of $T(r)$ with respect to r, the rate of change in time with respect to the interest rate using the quotient rule. (Note that $\ln(k)$ is a constant.)
3. Choose 5 interest rates between 2% and 20%. Complete the following table by calculating the values of $T(r)$ and $T'(r)$, correctly rounded to two decimal places. (Note that the rates do not necessarily have to be whole numbers. Examples such as 5.5% or 9.85% are acceptable.)

Interest Rates, *r*	$T(r)$	$T'(r)$

4. What can you say about the values that you calculated with the interest rates that you have chosen?
5. Show that your interpretation is correct by sketching the graph of $T(r)$ and $T'(r)$ using Excel or another graphing utility. (If necessary, the graphs may be pasted into a Word document and attached to the DB thread.)
6. Compare your calculations to any of your classmates'. Explain how different values for *k* and *r* affect your calculations.
7. Which intelli**path** Learning Nodes helped you with this assignment?

The following are the steps to derive the formula in this assignment from the formula for compounding interest at discrete periods of time, $A = P(1 + \frac{r}{n})^{nt}$, where *A* is the future amount, *P* is the principal, *r* is the decimal equivalent of the annual interest rate, *n* is the number of times compounded in 1 year, and *t* is the total time in years.

$$A = P(1 + \left(\frac{1}{12}\right)r)^{12t}$$

$$\frac{A}{P} = (1 + 0.0833r)^{12t}$$

$$\ln\left(\frac{A}{P}\right) = \ln(1 + 0.0833r)^{12t}$$

$$\ln\left(\frac{A}{P}\right) = 12t \times \ln(1 + 0.0833r)$$

$$t = \frac{\ln\left(\frac{A}{P}\right)}{12\ln(1 + 0.0833r)}$$

Let $k = \frac{A}{P}$, the ratio of the ending amount to the beginning amoutn, and replace *t* by $T(r)$ to indicate that this is a function of the decimal equivalent of the annual interest rate, *r*. Then, we get the following formula:

$$T(r) = \frac{\ln(k)}{12\ln(1 + 0.0833r)}$$

Notice that, from the derivation above, if the compounding is quarterly, the formula would be as follows:

$$T(r) = \frac{\ln(k)}{4\ln(1 + 0.025r)}$$

Compounding daily would be as follows:

$$T(r) = \frac{\ln(k)}{365\ln(1 + 0.00274r)}$$

Compounding annually would be as follows:

$$T(r) = \frac{\ln(k)}{\ln(1 + r)}$$