AMTH140 Assignment 4

This assignment covers second-order recurrence relations, recursion, and graph theory, including: trails, paths, circuits, graph isomorphism, and matrix representations of graphs.

Question 1: [*30 marks*]

Solve the following second-order linear homogeneous recurrence relations with constant coefficients.

(a) $a_n = -4a_{n-1} - 4a_{n-2}$ for all integers $n \geq 2$ with $a_0 = 0$, and $a_1 = -1$.

(b) $a_n = a_{n-1} + 6a_{n-2}$ for all integers $n \geq 2$ with $a_0 = 0$, and $a_1 = 3$.

Question 2: [5 *marks*]

Consider the following function $f : \mathbf{Z}^+ \to \mathbf{Z}$ for all positive integers:

$$f(n) = \begin{cases} n - 10 & \text{if } n > 100 \\ f(f(n+11)) & \text{if } n \leq 100 \end{cases}$$

Find $f(99)$, $f(100)$, and $f(101)$.

Question 3: [*30 marks*]

(a) For the graph in Figure 1, determine whether the following walks are trails, paths, closed walks, circuits, simple circuits, or just walks.

1. $v_1e_2v_2e_3v_3e_4v_4e_5v_2e_2v_1e_1v_0$
2. $v_2v_3v_4v_5v_2$
3. $v_4v_2v_3v_4v_5v_2v_4$
4. $v_2v_1v_5v_2v_3v_4v_2$
5. $v_0v_5v_2v_3v_4v_2v_1$
6. $v_5v_4v_2v_1$

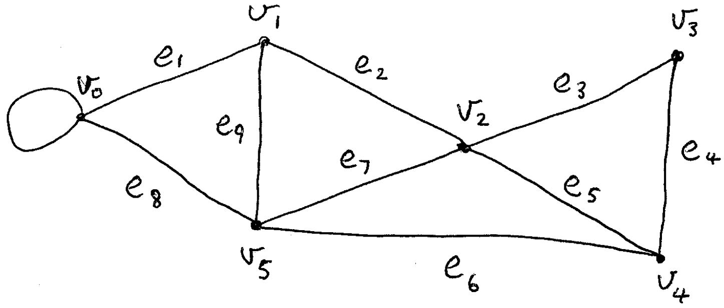

Figure 1: Graph for Question 3(a).

(b) Determine whether the graph in Figure 2 has an Euler circuit. If it does, describe one. If it does not, explain why not.

(c) Determine whether each graph in Figure 3 has a Hamiltonian circuit. If it does, describe one. If it does not, explain why not by making use of Proposition 10.2.6 on the existence of subgraphs of graphs with Hamiltonian circuits.

Question 4: *[20 marks]*

Consider the following adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

(a) Draw the graph corresponding to this adjacency matrix.

(b) How many walks of length 1, 2, and 3 are there from v_1 to v_1?

(c) How many walks of length 3 are there from v_2 to v_3?

Question 5: *[15 marks]*

Determine whether the two graphs in Figure 4 are isomorphic. If they are, give the vertex set mapping $g : V(G) \to V(G')$, and the edge set mapping $h : E(G) \to E(G')$ that define the isomorphism. If they are not, give an invariant for the graph isomorphism that they do not share.

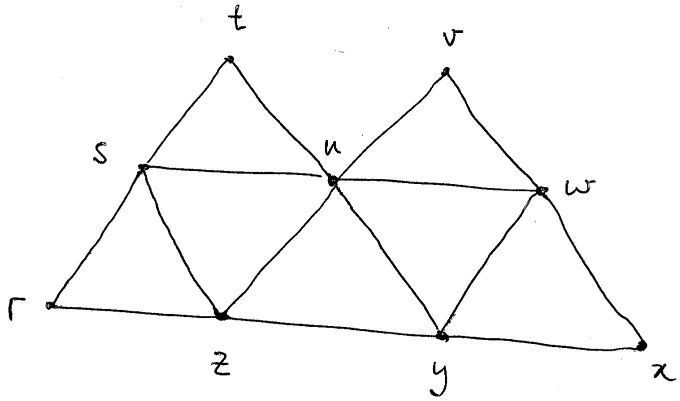

Figure 2: Graph for Question 3(b).

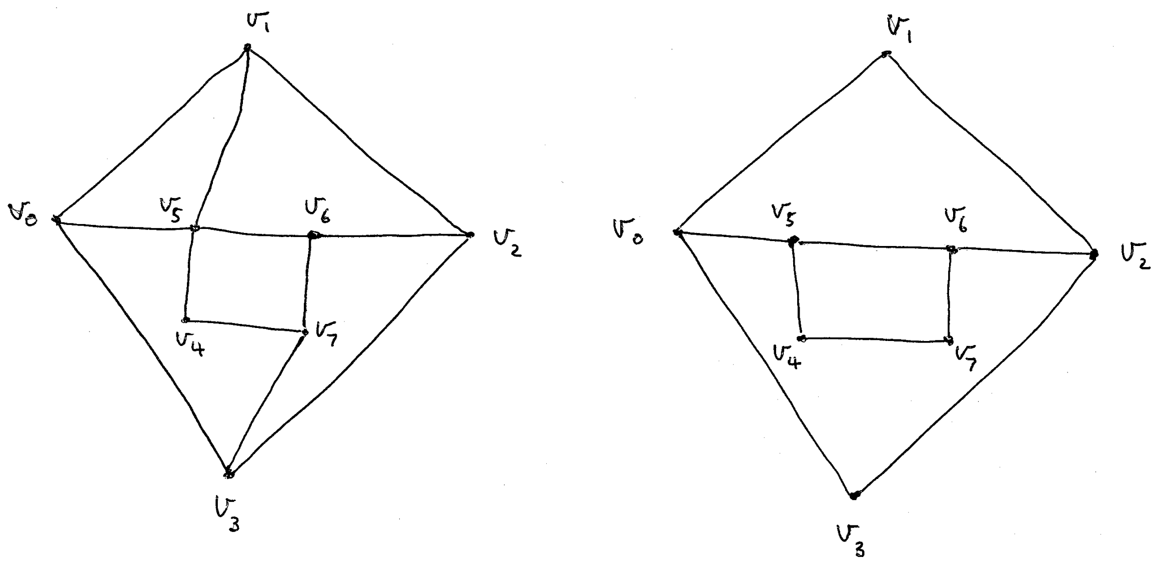

Figure 3: Graphs for Question 3(c).

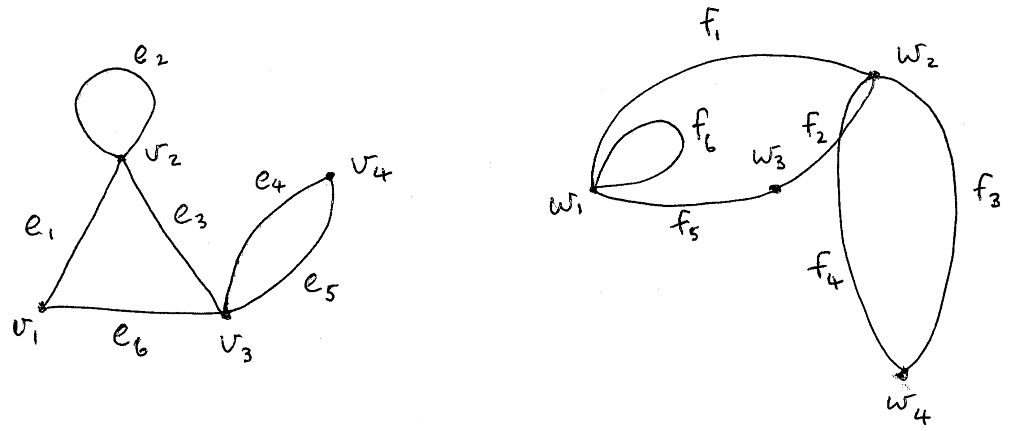

Figure 4: Graphs for Question 5.