**Math 520-Algebra I** **Assignment 1-Fall 2017** **Due: Sept. 12th**

Name:

1. Let $S$ be a nonempty set, $G$ is a group (written additively), and $M(S, G)$ the set of all functions $f : S \to G$. Define addition in $M(S, G)$ as follows: $(f + g) : S \to G$ is given be $(f + g)(s) = f(s) + g(s) \in G$. Prove that $M(S, G)$ is a group, and it is an abelian if $G$ is.

2. Write out a multiplication table for the group $D_4^*$, **the group of symmetries of the square**.

3. Suppose $G$ is a group. Prove that the following conditions are equivalent:
   (a) $G$ is abelian.
   (b) $(ab)^2 = a^2b^2$ for all $a$, $b \in G$.
   (c) $(ab)^{-1} = a^{-1}b^{-1}$ for all $a$, $b \in G$.

   [*Hint: Prove (a) $\Leftrightarrow$ (b) and (a)$\Leftrightarrow$ (c)*]

4. (a) Show that the relation given by $a \sim b \Leftrightarrow a - b \in \mathbb{Z}$ is an equivalence relation on the additive group $\mathbb{Q}$. [*Hint: Show that $\sim$ is reflexive, symmetric, and transitive*].
   (b) Show that the relation in (a) is a congruence relation on the additive group $\mathbb{Q}$. [*Hint: Show that if $a_1 \sim a_2$ and $b_1 \sim b_2$ then $(a_1 + b_1) \sim (a_2 + b_2)$*].

5. Write out an addition table for $\mathbf{Z}_2 \oplus \mathbf{Z}_2 \cdot \mathbf{Z}_2 \oplus \mathbf{Z}_2$ is called the **Klein four group**.

6. Lat $S$ be the set of all real numbers except $-1$. Define $*$ on $S$ by $a * b = a + b + ab$.
   (a) Show that $S$ is a group under the given binary operation $*$.
   (b) Find the solution of the equation $2 * x * 3 = 7$ in $S$.