CSE 2331 Homework 1
Autumn, 2017
(Total 75 points)

1. [30 points] Give the asymptotic complexity of each of the following functions in simplest terms. Your solution should have the form $\Theta(n^\alpha)$ or $\Theta((\log_\mu(n))^\beta)$ or $\Theta(n^\alpha(\log_\mu(n))^\beta)$ or $\Theta(\gamma^{\delta n})$ or $\Theta(1)$ where $\alpha, \beta, \gamma, \delta, \mu$ are constants. (**No need** to give any justification or proof.)

   (a) $f_a(n) = \log_2(3^{n+2} + 5n^3 + 1)$;
   (b) $f_b(n) = n^{0.1} \times \lg(4n^5 - 3n^3) + 3n^{0.2}$;
   (c) $f_c(n) = 3\log_4(4n+1) \times \log_3 n + \log_2(6n^2 + 8n)$;
   (d) $f_d(n) = 6^{13} + 2^6 \times 7\log_4(62)$;
   (e) $f_e(n) = 2(n+4)\log_3(2n^3+1) + 5n + \sqrt{2n}$;
   (f) $f_f(n) = 15^n - 10^n + n^{100}$;
   (g) $f_g(n) = 8^{\sqrt{n}} + 2^n$;
   (h) $f_h(n) = 3 \times 5^{n+9} + 6 \times 3^{n+9}$;
   (i) $f_i(n) = \sqrt{2n^3 + 3n^2}$;
   (j) $f_j(n) = 9 \times 2^{\log_2(n^2+2n)}$;
   (k) $f_k(n) = (3\log_4(n^2+8) + 6\sqrt{n}) \times (\log_5 n + 4\log_3 n)$;
   (l) $f_l(n) = 3^{4n} + 4^{3n}$;
   (m) $f_m(n) = 5\log_{10}(7n^3 - 6n + 9) + 9\log_2(5n^{4.5} + 33n)$;
   (n) $f_n(n) = (4n^3 + 2n^2 + 1) * (n^2 + 5n + 13) * (12n - 6)$;
   (o) $f_o(n) = \log_{10}(4^n + 6^n + 8^n)$;

2. [20 points]Rank the following functions by order of growth: that is, find an arrangement $g_1, g_2, \ldots,$ of these functions such that either $g_i = O(g_{i+1})$ or $g_i = \Theta(g_{i+1})$, for any two consecutive functions g_i and g_{i+1} in your ordered list. (For example, given $n, 2, 2^n, 3n + \sqrt{n}$, your answer will look like: $2 = O(n), n = \Theta(3n + \sqrt{n})$, and $3n + \sqrt{n} = O(2^n)$.) You need to make your answer as tight as possible, that is, you should say $g_i = \Theta(g_{i+1})$ whenever possible. In what follows, lg refers to $\log_2$, and ln refers $\log_e$ where e is the natural number. (**No need** to provide justificaiton and proof for your answers.)

$$n\lg n,\quad 2^{n+9},\quad \sqrt{2n^2\lg n + 3n},\quad 2^{\lg n},\quad \lg(n!),\quad n\sqrt{\ln n},\quad 5^{900},\quad \frac{1}{2}\cdot 2^n,\quad 2\cdot 3^n,\quad n\cdot 2^n,$$

$$n^{0.7},\quad \lg(6n+7)\times\lg(5n^{0.3}+21),\quad \sqrt{n^3-2n^2},\quad 3^{2n},\quad \log_6((2n+4)(3n+2)(5n+6)),\quad 2^{\ln n}.$$

3. [10 points]Specify whether each of the following statement is true or false. If it is true, prove it. If it is false, disprove it by providing a counter example. (All functions below are **positive functions**.)

   (a) If $f(n) = O(g(n))$, then $f^2(n) = O(g^2(n))$ (where $f^2(n) = f(n) * f(n)$ and $g^2(n) = g(n) * g(n)$).
   (b) $f(n) + g(n) = \Theta(min\{f(n), g(n)\})$.

4. [10 points] Provide an example in each of the following case, and briefly justify your answer.

   (a) Give an example of a function $f(n)$ such that: $f(n) \in O(\sqrt{n})$ and $f(n) \in \Omega(\log n)$ but $f(n) \notin \Theta(\sqrt{n})$ and $f(n) \notin \Theta(\log n)$.
   (b) Give an example of a function $f(n)$ such that: $f(n) \in O(\frac{1}{2}\sqrt{n}\log n)$ and $f(n) \in \Omega(100\sqrt{n})$ but $f(n) \notin \Theta(\frac{1}{2}\sqrt{n}\log n)$ and $f(n) \notin \Theta(100\sqrt{n})$.

5. [5 points] Prove that $7\sqrt{3n^5 - 9n^3 + 2} \in \Theta(n^{2.5})$.