University of Maryland University College
CMSC 150 – Introduction to Discrete Structures

Final Examination

1. (10 pts, 2 parts – 5 points each) For each the following groups of sets, determine whether they form a partition for the set of integers. Explain your answer.

a. $A_1 = \{n \in \mathbf{Z} : n > 0\}$

$A_2 = \{n \in \mathbf{Z} : n < 0\}$

b. $B_1 = \{n \in \mathbf{Z} : n = 2k$, for some integer $k\}$

$B_2 = \{n \in \mathbf{Z} : n = 2k + 1$, for some integer $k\}$

$B_3 = \{n \in \mathbf{Z} : n = 3k$, for some integer $k\}$

2. (10 pts, 4 parts – 2.5 points each) Define f: $\mathbf{Z} \rightarrow \mathbf{Z}$ by the rule $f(x) = 6x + 1$, for all integers x.

a. Is f onto?

b. Is f one-to-one?

c. Is it a one-to-one correspondence?

d. Find the range of f.

Explain each of your answers.

3. (10 pts, 2 parts – 5 points each) f: $\mathbf{R} \rightarrow \mathbf{R}$ and g: $\mathbf{R} \rightarrow \mathbf{R}$ are defined by the rules:

$f(x) = x^2 + 2 \; \forall \; x \in \mathbf{R}$

$g(y) = 2y + 3 \; \forall \; y \in \mathbf{R}$

Find $f \circ g$ and $g \circ f$.

4. (10 pts, 3 parts – 3.33 points each) Determine whether the following binary relations are reflexive, symmetric, antisymmetric and transitive:

a. $x \mathrel{R} y \Leftrightarrow xy \geq 0 \; \forall x, y \in \mathbf{R}$

b. $x \mathrel{R} y \Leftrightarrow x > y \; \forall x, y \in \mathbf{R}$

c. $x \mathrel{R} y \Leftrightarrow |x| = |y| \; \forall x, y \in \mathbf{R}$

For each of the above, indicate whether it is an equivalence relation or a partial order. If it is a partial order, indicate whether it is a total order. If it is an equivalence relation, describe its equivalence classes.

5. (10 pts) Determine whether the following pair of statements are logically equivalent. Justify your answer using a truth table.

$p \to (q \to r)$ and $p \wedge q \to r$

6. (10 pts, 2 parts – 5 points each) Prove or disprove the following statement:

$\forall\, n, m \in \mathbb{Z}$, If n is even and m is odd, then $n + m$ is odd

Then write the negation of this statement and prove or disprove it.

7. (10 pts) Prove the following by induction:

$$\sum_{i=1}^{n} 3i - 2 = \frac{3n^2 - n}{2}$$

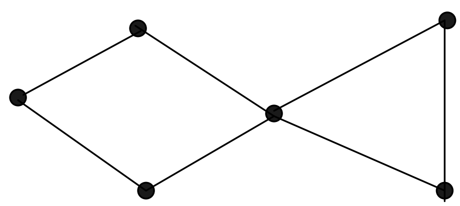
8. (10 pts) Use the permutation formula to calculate the number permutations of the set {V, W, X, Y, Z} taken three at a time. Also list these permutations.

9. (10 pts, 2 parts – 5 points each) Translate the following English sentences into statements of predicate calculus that contain double quantifiers and explain whether it is a true statement.

a. Every rational number is the reciprocal of some other rational number.

b. Some real number is bigger than all negative integers.

10. (10 pts, 5 parts, 2 points each) Consider the following graph:



In each case, answer the question and then write the rationale for your answer.

a. Is this graph connected?

b. Is this a simple graph?

c. Does this graph contain any cycles?

d. Does this graph contain an Euler cycle?

e. Is this graph a tree?