## Assignment 1 - Boolean Algebra and Simplification and Digital Circuits
## Due: Monday July 8, 2017 11:59pm

Write or type your answers clearly and upload a PDF file to D2L by the deadline. Please see the syllabus for late policies on homework assignments.

1. [1 pt each - 5 pts total] Write a Boolean equation in sum-of-product forms for each of the following truth tables.

(a)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(b)

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(c)

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(d)

| A | B | C | D | Y |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

(e)

| A | B | C | D | Y |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

2. [1 pt each - 5 pts total] Write a Boolean equation in product-of-sums forms for each of the truth tables above.

3. [2 pt each - 4 pts total] Prove DeMorgan's theorem for three variables using truth tables:

   A. ( A + B + C )' = A' B' C'
   B. ( A B C )' = A' + B' + C'

4. Simplify the following Boolean expressions to the minimum number of terms using the properties of Boolean algebra (show your work and write the property you are applying).

   A. [1 pts] XY + XY'
   B. [1 pts] (X + Y)(X + Y')
   C. [1 pts] (A + B)' (A' + B')'
   D. [1 pts] ABC + A'B + ABC'
   E. [2 pts] XY + X(WZ + WZ')

5. [2 pts each - 10 pts total] Draw the circuit logic diagrams for both the original and simplified expressions from Problem 2.

   **example solution**
   original expression:    F = A'C' + ABC + AC'
   simplified expression:  $F_{simplified}$ = AB + C'
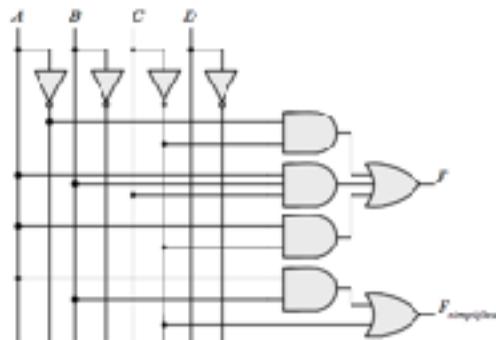


6. For the following functions, write out the truth table and write the function in sigma form, ie. F = $\sum (1,3,4)$

   A. [2 pts] F = XY + XY' +X'Y
   B. [2 pts] F = X'Z' + YX

7. Simplify the following, using three-variable K-maps.
   C.  [2 pts] F(X,Y,Z) = $\sum (0,1,2,3,7)$
   D.  [2 pts] XY + X'Y'Z' + X'YZ'
   E.  [2 pts] X'Y' + YZ + X'YZ'

   **example solution**
   F(x,y,z) = $\sum (3,5,6,7)$
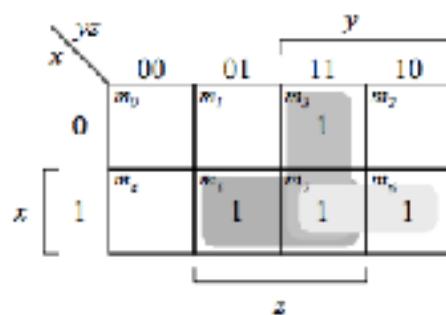
$$F_{\text{simplified}} = xy + xz + yz$$

8.  Simplify the following using K-maps.
    A.  [3 pts] $F(A,B,C,D) = \sum (4,6,7,15)$
    B.  [3 pts] $F(A,B,C,D) = \sum (2,3,12,13,14,15)$
    C.  [3 pts] A'B'C'D' + AC'D' + B'CD' + A'BCD + BC'D

9.  [3 pts] Design a circuit with three inputs (x,y,z) and one output that outputs
    true if the binary value of the inputs is less than 3. Construct the truth table,
    simplify using a K-map, and draw out the logic circuit diagram.

10. [3 pts] Design a circuit with three inputs (x,y,z) and one output that outputs
    true if the binary value of the inputs is an odd number. Construct the truth
    table, simplify using a K-map, and draw out the logic circuit diagram.

11. [2 pts] Explain the difference between truth table, state table, characteristic
    table, and excitation table, in a few concise sentences.

12. [1 pts] The contents of a 4-bit register are initially 1011. The register is
    shifted six times to the right, with the serial input being 110110. What are the
    contents of the register after each shift?

13. [4 pts] Implement a 10x1 MUX using 4x1 MUX. It should take selection
    codes 0000 to 1001.

14. [2 pts] Given the following S and R input waveform, sketch the output
    waveform Q, of an SR latch.

15. [2 pts] Given the following input waveforms, sketch the output waveform Q, of a D flip-flop.



16. Is the following circuit an example combinational logic or sequential logic? *Briefly (in a few sentences or less), d*escribe the behavior, in terms of the relationship between the inputs and outputs.
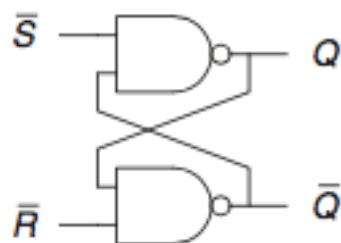


17. [**EXTRA CREDIT - 6 pts**] Design a four-bit 2's complementer circuit (ie. takes a 4-bit input and outputs it's 2's complement - negate each bit and add 1). Show your work for each output bit. Simplify each output using XOR gates.