Concordia University
Comp 232-AA Assignment One

Name:
Student number:

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Due: 2017/05/18

Show all work on these sheets. In proofs use formats employed in the completed Note Outlines (Sec 1.1-1.8) done in class. If not indicated in question text, state the type of proof being used.

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[04] **1.** Write each statement and its indicated form as: If _____ then _____

(i) n^2 is greater than 9 only if n is greater than 3

statement: _____

converse: _____

(ii) $x \geq 0$ is necessary for $|x| = x$

statement: _____

contrapositve: _____

(ii) $x \geq 0$ is sufficient for $|x| = x$

statement: _____

inverse: _____

(iii) I will drve to work unless it does not rain.

statement: _____

inverse of contrapositve: _____

[05] **2.** Knights always tell the truth, Knaves always tell a lie and Spies can tell the truth or a lie. You meet three people A, B, C one of each type. Each of A, B, C knows what the other types are.
A says "I am the Knight", B says "I am the Knave", C says "I am not the knave". Let p represent A is the Knight, q represent B is the Knight and r represent C is the Knight. Use a truth table to assist in the determination of the type of each person.

[06] **3.** Simplify each statement using logical equivalences then state whether each statement is a Tautology, Contradiction or Contingency. If statement is a Contingency state one set of values for p, q that make statement T and one set that makes it F.

a) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

b) $(p \rightarrow q) \wedge (q \rightarrow \neg p) \wedge (p \vee q)$

[05] **4.** $P(x)$ is statement " Student x knows Calculus", $Q(y)$ is statement "Class y contains a student who knows Calculus". Express each statement below using quantifiers for P, Q

a) Some students know Calculus _____

b) Not every student knows Calculus _____

c) Every student in every class knows Calculus _____

d) Every class contains a student who knows Calculus _____

e) There is at least one class with no students who know Calculus _____

[06] **5.** $P(x, y)$ means $3xy + x = 7$, where x, y are Real numbers. Determine the truth value of each statement below and justify your answer.

a) $\forall x \exists y P(x, y)$

b) $\neg \forall x \exists y \neg P(x, y)$

[06] **6.** Determine if each equivalence is valid or invalid. If valid justify, if invalid list a counter example.

a) $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

b) $\forall x(P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall x Q(x)$

c) $\forall x(P(x) \iff Q(x)) \equiv \forall x P(x) \iff \forall x Q(x)$

[04] **7.** For this question form solutions using Boolean Algebra notation instead of $\wedge, \vee, \neg$, T, F. Consider the decision table:

p	q	r	
False	False	False	_____
False	False	True	_____
False	True	False	_____
False	True	True	_____

a) Beside each row write the Conjunction expression in terms of p, q, r

b) Write the Disjunction of the four answers in a) _____

c) Evaluate the simplified Boolean Algebra expression for the answer in b)

[06] **8.** Consider the Theorem: If $3n^2 + 8$ is even then n is even. Prove this Theorem three ways:

a) Proof by Contraposition:

b) Direct proof:

c) Proof by Contradiction:

[03] **9.** Given a group of 40 people prove that at least four of these people were born in the same month.

[03] **10.** Prove $\forall x, x \neq 0, x \epsilon R : x^4 + \frac{9}{x^4} \geq 6$ Hint: Use backword reasoning to discover an equation to start a Direct proof.

[03] **11.** The notation $max(x, y)$, where $x, y \epsilon R, x \neq y$ implies the greater of values x, y. Use a proof by cases to prove $max(a, max(b, c)) = max(max(a, b), c)$ where $a, b, c \epsilon R, a \neq b \neq c$

[09] **12** **a)** Prove that: If a is an even integer then $a^n, n\epsilon\, Z^+$ is an even Integer.

b) Prove that: If a is an odd integer then $a^n, n\epsilon\, Z^+$ is an odd Integer.

c) Assuming the results in a) and b prove that $\log_3 4$ is Irrational.

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total

60 ***marks***