3. A random sample of 144 patients, suffering from a particular disease are given a new medicine. 108 of the patients report an improvement in their condition.

a) Construct a 90% (two-sided) confidence interval for the overall improvement rate of this medicine.

b) Find the minimum sample size required if we want to estimate the improvement rate of this medicine to within 4% with 90% confidence if it is known that the improvement rate is between 70% and 90%.

c) Find the minimum sample size required if we want to estimate the improvement rate of this medicine to within 4% with 90% confidence if we do not make any assumptions about the improvement rate.

d) The company that manufactures this medicine claims 80% improvement rate. We suspect that it is less than 80%. Use $\alpha = 0.05$ to perform the appropriate test. State the null and alternative hypothesis for this test in terms of the relevant parameter, report the value of the test statistic, the critical value(s), and state your decision.

e) Find the p-value (approximately) of the test in part (d).

f) Suppose the actual value of the overall improvement rate of this medicine is 77%. Then in part (d) (correct decision, Type I Error, Type II Error) was made.