PROCTOR'S STATEMENT

This is to certify that _____ wrote an examination in the course _____**Math 208**_____ under my personal supervision and received no outside aid from any source whatsoever. The student was verified through a picture ID prior to taking the examination. The completed examination is being sent to the Online and Distance Education Office by me.

_____
Signature of Examination Proctor

_____
Position

_____
Date

Math 208 Correspondence Lesson 7: Exam 1 Name: _____

TIME LIMIT: 2 hours; no books, no notes, no calculators, etc

I: ($2\frac{1}{2}$ points each) True or False (circle your answer):

1) True False: The sentence *There is life on Mars* is a proposition.

2) True False: $\neg(p \longrightarrow q) \equiv p \longrightarrow \neg q$

3) True False: $\forall x \exists y P(x, y) \equiv \exists y \forall x P(x, y)$

4) True False: $s \longrightarrow t$ *and* $\neg s$, $\therefore \neg t$ is a valid rule of inference.

5) True False: Every element of the empty set is bigger than 2.

6) True False: The set $\{1, \{1, 2\}, \{2\}\}$ has two elements.

7) True False: For all sets A, B, we have $A \cap B \subseteq A \cup B$.

8) True False: In an indirect proof of $H \longrightarrow C$ it is OK to use $\neg C$ as a step.

9) True False: (cat, a) satisfies the relation $C(x, y)$: x *uses letter* y.

10) True False: The relation on the set of all people $S(x, y)$: x *and* y *have met* is transitive.

11) True False: If $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are each one-to-one functions, then $g \circ f : A \longrightarrow C$ is also one-to-one.

12) True False: The relation $G(x, y)$ *x lives within one mile of y* is an equivalence relation on the set of all people.

II: (5 points each) Multiple Choice (circle your answer):

1) The converse of $u \longrightarrow v$ is

(a) $\neg v \longrightarrow \neg u$

(b) $\neg v \longrightarrow u$

(c) $\neg u \longrightarrow v$

(d) $v \longrightarrow u$

(e) $\neg u \longrightarrow \neg v$

2) $\neg\exists x \forall y\, P(x, y)$ is logically equivalent to

(a) $\forall x \exists y\, \neg P(x, y)$

(b) $\exists x \forall y\, \neg P(x, y)$

(c) $\exists y \forall x\, P(x, y)$

(d) $\forall x \exists y\, \neg P(y, x)$

(e) $\forall x \neg \exists y\, P(x, y)$

3) If the set $A \times B$ has 45 elements and the set A has 3 elements, then the number of elements in B is

(a) 5

(b) 10

(c) 15

(d) 20

(e) 25

4) The first line in a proof of THM: *If* B *is a blisk, then* B *is a glumx.* is *Suppose* B *is a blisk, and* B *is not a glumx.* The method of proof being used is:

(a) Proof by Contradiction

(b) Direct Proof

(c) Indirect Proof

(d) Proof by Cases

(e) Proof by Confusion

5) R is the relation on the integers given by $R(n, m)$: $n - m$ ends with 0. For example, $R(12, 102)$ is true since $12 - 102 = -90$, and -90 does end with a 0. But $55 - 11 = 44$ so $R(55, 11)$ is false since 44 does not end with a 0. Circle all the properties the relation R has in the list below.

(a) Reflexive

(b) Symmetric

(c) Antisymmetric

(d) Transitive

(e) Self Referential

6) Let $A = \{1, 2\}$ and $B = \{a, b\}$. The number of different functions from A to B is

(a) 1

(b) 2

(c) 3

(d) 4

(e) Infinitely many.

7) Let T be the set of all English words with two or more letters. Define the function s from the set T to the set of 26 letters by the rule: (w) is the second letter in the word w. For example $s(cat) = a$. Circle all the true statements in the list below.

(a) s is one-to-one.

(b) s is onto.

(c) s has an inverse function.

(d) $s(to) = s(too)$

(e) None of the above statements are true.

8) Let C be the relation on the cards in the usual 52 card deck defined by $C(x, y)$: x *and* y *have the same suit.* Circle all the true statements in the list below.

(a) The equivalence class, [3♠], of the 3♠ contains 13 elements.

(b) The equivalence class, [3♠], of the 3♠ contains 4 elements.

(c) [3♠] = [4♠].

(d) [3♠] = [3♣]

(e) None of the above statements are true.

III. (10 points each) Problems

Do any three of the following four problems. If you do all four, I'll count your best three.

1) Use a truth table to determine if $\neg p \vee (q \longrightarrow r)$ is logically equivalent to $r \vee \neg(p \longrightarrow q)$.

2) Express the following argument in symbolic form and show, by applying rules of inference, that the argument is valid. Give the reason for each step in your proof.

If Ralph goes to graduate school, then he will be a professor. If Ralph doesn't go to graduate school, he will have to get a real job. If Ralph gets a real job, he will need a new suit. So, if Ralph doesn't become a professor, then he will need a new suit.

3) Give a direct proof that the product of two odd integers is odd.

4) Let $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the set of nonzero digits. Let S be the relation for x and y in D defined by $S(x, y)$: x *and* y *have the same number of letters in their names.* For example, 5S9 is true since 5 and 9 have the same number of letters in their names (namely four letters). Explain why S is an equivalence relation and list the elements of the equivalence class of 2.